HOpenGL – 3D Graphics with Haskell A small Tutorial (Draft)

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Publish early and publish often. That is the reason why you can read this. I started playing around with *HOpenGL* the Haskell port of OpenGL a common library for doing 3D graphics. I more or less took minutes of my efforts and make them public in this tutorial. I did not have any prior experience in graphics programming, when I started to work with HOpenGL.

The source of this paper is an XML-file. The sources are processed by an XQuery processor, XSLT scripts and $I\!\!AT_{\!E\!}X\,$ in order to produce the different formats of the tutorial.

I'd like to thank Sven Panne¹, the author of HOpenGL, who has been so kind to comment on first drafts of this tutorial.

 $^{^1 {\}rm Similar}$ name different person.

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Chapter 1

Introduction

In this chapter some basic background information can be found. You you can read the sections of this chapter in an arbitrary order. Whatever your personal preference is.

1.1 A Little Bit of Practice

Before you read a lot of technical details you will probably like to see something on your screen. Therefore you find some very simple examples in the beginning. This will give you a first impression, of how an OpenGL program might look like in Haskell.

1.1.1 Opening Windows

OpenGL's main purpose is to render some graphics on a device. This device is generally a window on your computer screen. Before you can draw something on a screen you will need to open a window. So let's have a look at the simpliest OpenGL program, which just opens an empty window:

```
_ HelloWindow.hs _
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
з
   main = do
4
      getArgsAndInitialize
\mathbf{5}
      createAWindow "Hello Window"
6
      mainLoop
7
8
    createAWindow windowName = do
9
      createWindow windowName
10
      displayCallback $= clear [ColorBuffer]
11
```

The first two lines import the necessary libraries. The main function does three things:

• initialize the OpenGL system

- define a window
- start the main procedure for dispaying everything and reacting on events

For the definition of a window with a given name we do two things:

- create some window with the given name
- define, what is to be done, when the window contents is to be displayed. In the simple example above we simply clear the screen of any color by filling it with the default background color.

This 10 lines can be compiled with ghc. Do not forget to specify the packages, which contain the OpenGL library. It suffices to include the package GLUT, which automatically forces the inclusion of the package OpenGL. GLUT is the graphical user interface, which comes along with OpenGL, i.e. the window managing system etc.

```
sep@swe10:~/hopengl/examples> ghc -package GLUT -o HelloWindow HelloWindow.hs
sep@swe10:~/hopengl/examples> ./HelloWindow
```

When you start the program, a window will be opened on your desktop. As you may have noticed, we did not specify any attribute of the window, like its size and position. GLUT is defined in a way that initial default values are used for unspecified attributes.

1.1.2Drawing into Windows

The simple program above did just open a window. The main purpose of OpenGL is to define some graphics which is rendered in a window. Before starting to systematically explore the OpenGL library let's have a look at two examples that draw something into a window frame.

Some Points

1

2 з

7

First we will draw some tiny points on the screen. We use the same code for openening some window:

SomePoints.hs

```
import Graphics.UI.GLUT
   import Graphics.Rendering.OpenGL
   main = do
4
      (progName,_) <- getArgsAndInitialize</pre>
\mathbf{5}
     createAWindow progName
6
     mainLoop
```

The only thing that has changed, is that we make use of one of the values returned by getArgsAndInitialize: the name of the program.

For the window definition we use the code from HelloWindow.hs. But instead of clearing the screen, when the window is to be displayed, we use an own display function:

```
SomePoints.hs
```

```
    s createAWindow windowName = do
    9 createWindow windowName
    10 displayCallback $= displayPoints
```

We want to draw some points on the screen. So let's define some points. We can do this in a list. Points in a three dimensional space are triples of coordinates. We can use floating point numbers for coordinates in OpenGL.

Eventually we need the display function, which displays these points.

```
SomePoints.hs

displayPoints = do

clear [ColorBuffer]

renderPrimitive Points

$mapM_ (\(x, y, z)->vertex$Vertex3 x y z) myPoints
```

As you see, when the window ist displayed, we want first everything to be cleared from the window. Then we use the HOpenGL function renderPrimitive. The first argument Point specifies what it is that we want to render; points in our case. For the second argument we need to transform our coordinates into some data, which is used by HOpenGL. Do not yet worry about this transformation.

As before, you will notice that again for quite a number of attributes we did not supply explicit values. We did not specify the Color of the points to be drawn. Moreover we did not define the coordinates of the graphics window. Looking at its result it is obviously a two dimensional view, where the lower left corner seems to have coordinates (-1,-1) and the upper right corner the (1,1). These values are default values chosen by the OpenGL library.

A Polygon

The points in the last section were rather boring? By changing a single word, we can span an area with these points. Instead of saying render the following as points, we can tell HOpenGL to render them as a polygon.

Example:

So here the program from above with one word changed. Points becomes Polygon.

```
APolygon.hs
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
^{2}
3
    main = do
4
       (progName,_) <-getArgsAndInitialize</pre>
\mathbf{5}
       createAWindow progName
6
       mainLoop
7
8
    createAWindow windowName = do
9
       createWindow windowName
10
       displayCallback $= displayPoints
11
12
13
    displayPoints = do
      clear [ColorBuffer]
14
      renderPrimitive Polygon
15
        mapM_ ((x, y, z)->vertex$Vertex3 x y z) myPoints
16
17
    myPoints :: [(GLfloat,GLfloat,GLfloat)]
18
    myPoints =
19
      [(-0.25, 0.25, 0.0)]
20
      ,(0.75, 0.35, 0.0)
^{21}
      ,(0.75, -0.15, 0.0)
^{22}
      ,((-0.75), -0.25, 0.0)]
23
```

The resulting window can be found in figure 1.1.

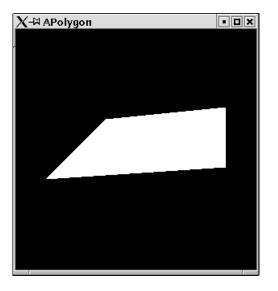


Figure 1.1: A simple polygon.

1.2 A Little Bit of Theory

1.2.1 Haskell

Haskell [?] is a lazily evaluated functional programming language. This means that there are no mutable variables. A Haskell program consists of expressions, which do not have any side effects. Expressions are only evaluated to some value when this is absolutely necessary for program execution. This means it is hard to predict in which order subexpressions get evaluated.

Expressions evaluate to some value without changing any state. This is a nice property of Haskell, because it makes reasoning about programs easier and programs are very robust.

1.2.2 OpenGL

OpenGL on the other hand is a graphics library which is defined in terms of a state machine. A mutable state modells the current state of the world. Functions are executed one after another on this state in order to modify certain variables. E.g. one variable keeps the current color to which all drawing statements refer. There is a statement which allows to set the color variable to some other value.

A comprehensive introduction to OpenGL can be found in the so called *redbook*[WBN⁺97]. OpenGL comes along with a utility library called GLU [CFH⁺98] and a system independent GUI library called GLUT [Kil96].

1.2.3 Haskell and OpenGL

Having said this, Haskell and OpenGL seem to cooperate badly. There seems to be a great mismatch between the fundamental concepts of the two. However, the designers of Haskell discovered a very powerful structure, which is a perfect concept for modelling state changing functions in a purely functional language: *Monads*[Wad90]. Most Haskell programmers do not worry about the theory of monads but simply use them, whenever they do I/O, state changing functions or in parser construction. With monads functional programs can almost look like ordinary imperative programs [PJW93].

Monads are so essential to functional programming, that they have a special syntactic construct in Haskell, the *do notation*.

Consider the following simple Haskell program, which uses monads:

Print.hs

```
main = do
  let x = 5
  print x
  let x = 6
  print x
  xs <- getLine
  print (length xs)</pre>
```

The monadic statements start with the keyword do. The statements have side effects. Variables can be defined and redefined in let-expressions¹. Monadic statements can have a result. This can be retrieved from the statement by the <- notation.

On another aspect OpenGL and Haskell perfectly match. In OpenGL functions are assigned to different data objects, e.g. a display function is passed to windows. Since functions are first class citizens, they can easily and type safe be passed around².

1.3 A Little Bit of Technics

If you want to start programming OpenGL in Haskell you need to be one of the brave, who compile sources from the functional programming CVS repository in Glasgow. There is not yet a precompiled version of the current HOpenGL library. Go to the website (www.haskell.org/ghc) of the Glasgow Haskell Compiler (GHC), follow closely the instructions on the page *CVS cheat sheet*. When doing the ./configure step, then use the option --enable-hopengl. i.e. start the command ./configure --enable-hopengl. This will ensure that the Haskell OpenGL library will be build and the packages OpenGL and GLUT are added to your GHC installation.

To compile Haskell OpenGL programs you simply have to add the package information to he command line invocation of GHC, i.e. use: ghc -package GLUT MyProgram.hs

Everything else, linking etc is done by GHC. You do not have to worry about library paths or anything else.

1.4 A Little Bit of History

The Haskell port of OpenGL has been done by Sven Panne. Currently a stable version exists and can be downloaded as precompiled binary. This tutorial deals with the completely revised version of HopenGL, which has a more Haskell like API and needs less technical overhead. This new version is not yet available as ready to use package. You need to compile it yourself.

This tutorial has been written with no prior knowledge of OpenGL and no documentation of HOpenGL at hand.

For the *old* version 1.04 of HOpenGL an online tutorial written by Andre W B Furtado exists at (www.cin.ufpe.br/~haskell/hopengl/index.html).

¹Variables bound in let-expressions are not variables as known from imperative languages. Line 4 in the example above does not assign a new value to a variable x but defines a new variable x.

 $^{^{2}}$ Unlike the object orientated languages Java, which misses an easy way to pass functions around.

Chapter 2

Basics

2.1 Setting and Getting of Variables

From what we have learnt in the introduction, we know that we are dealing with a state machine and will write a sequence of monadic functions which effect this machine. Before we start drawing fancy pictures let us explore the way values are set and retrieved in HOpenGL.

2.1.1 Setting values

The most basic operation is to assign values to variables in the state machine. In HOpenGL this is done by means of the operator \$=1 You do not need to understand, how this operator is implemented. You simply can imagine that it is an assignment operator. The left operand is a variable which gets assigned the right operand. We can revisit the first program, which simply opened a window.

Example:

When we have created a window, we assign a size to it:

```
_ Set.hs
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
3
    main = do
4
      getArgsAndInitialize
\mathbf{5}
      myWindow "Hello Window"
6
      mainLoop
7
8
    myWindow name = do
9
      createWindow name
10
      windowSize $= Size 800 500
11
      displayCallback $= clear [ColorBuffer]
12
```

 $^{^{1}}$ A nicer choice for this operator would have been :=, but this is not allowed for a function operator in Haskell, but denotes an infix constructor.

One example of the assignment operator we have allready seen. In the last line we assign a function to the variable displayCallback. This function will be executed, whenever the window is displayed.

As you see, more you do not need to know about **\$=**. But if you want to learn more about it read the next section.

Implementation of set

The operator **\$=** is defined in the module Graphics.Rendering.OpenGL.GL.StateVar as a member function of a type class:

```
infixr 2 $=
    class HasSetter s where
        ($=) :: s a -> a -> IO ()
```

The variables of HOpenGL, which can be set are of type SettableStateVar e.g.: windowTitle :: SettableStateVar String. Further variables that can be set for windows are: windowStatus, windowTitle, iconTitle, pointerPosition,

2.1.2 Getting values

You might want to retrieve certain values from the state. This can be done with the function get, which is in a way the corresponding function to the operator =.

_ Get.hs _

Example:

You can retrieve the size of the screen:

```
import Graphics.UI.GLUT
import Graphics.Rendering.OpenGL
main = do
getArgsAndInitialize
x<-get screenSize
print x</pre>
```

When you compile and run this example the size of your screen it printed:

```
sep@swe10:~/hopengl/examples> ghc -package GLUT -o Get Get.hs
sep@swe10:~/hopengl/examples> ./Get
Size 1024 768
sep@swe10:~/hopengl/examples>
```

Implementation of get

There is a corresponding type class, which denotes that values can be retrieved from a variable:

1

2

```
class HasGetter g where
get :: g a -> IO a
```

Variables which implement this class are of type GettableStateVar a.

2.1.3 Getting and Setting Values

For most variables you would want to do both: setting them and retrieving their values. These variables implement both type classes and are usually of type: StateVar.

But things do not always work so simple as this sounds.

Example:

The following program sets the size of a window. Afterwards the variable windowSize is retrieved:

```
SetGet.hs
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
3
   main = do
4
      getArgsAndInitialize
\mathbf{5}
      myWindow "Hello Window"
6
      mainLoop
7
8
    myWindow name = do
9
      createWindow name
10
      windowSize $= Size 800 500
11
      x<-get windowSize
12
      print x
13
      displayCallback $= clear [ColorBuffer]
14
```

Running this program gives the somehow surprising result:

sep@swe10:~/hopengl/examples> ./SetGet
Size 300 300

The window we created, has the expected size of (800,500) but the variable windowSize still has the default value (300,300).

The reason for this is, that setting the window size state variable has not a direct effect. It just states a wish for a window size. Only in the execution of the function mainLoop actual windows will be created by the window system. Only then the window size will be taken into account. Up to that moment the window size variable still has the default value. If you print the window size state within some function which is executed in the main loop, then you will get the actual size. By the way: you can try initialWindowSize without getting such complecated surprising results.

2.1.4 What do the variables refer to

The state machine contains variables and stacks of objects, which are effectedly mutated by calls to monadic functions. However not only the get and set statements modify the state but also statements like **createWindow**. This makes it in the beginning a bit hard to understand, when the state is changed in which way.

The createWindow statement not only constructs a window object, but keeps this new window as the current window in the state. After the createWindow statement all window effecting statements like setting the window size, are applied to this new window object.

2.2 Basic Drawing

2.2.1 Display Functions

There is a window specific variable which stores the function that is to be executed whenever a window is to be displayed, the variable displayCallback. Since Haskell is a higher order language, it is very natural to pass a function to the assignment operator. We can define a function with some arbitrary name. The function can be assigned to the variable displayCallback. In this function we can define a sequence of monadic statements.

Clearing the Screen

A first step we would like to do whenever the window needs to be drawn is to clear from it whatever it contains². HOpenGL provides the function clear, which does exactly this job. It has one argument. It is a list of objects to be cleared. Generally you will clear the so called color buffer, which contains the color displayed for every pixel on the screen.

Example:

The following simple program opens a window and clears its content pane whenever it is displayed:

_ Clear.hs _

```
import Graphics.UI.GLUT
1
^{2}
    import Graphics.Rendering.OpenGL
3
    main = do
4
      (progName,_) <- getArgsAndInitialize</pre>
5
      createAWindow progName
6
      mainLoop
7
8
    createAWindow windowName = do
9
      createWindow windowName
10
      displayCallback $= display
11
12
    display = clear [ColorBuffer]
13
```

²Otherwise you might see arbitrary parts of other applications in your window frame.

First Color Operations

The window in the last section has a black background. This is because we did not specify the color of the background and HOpenGL's default value for the background color is black. There is simply a variable for the background color.

For colors several data types are defined. An easy to use one is:

```
data Color4 a = Color4 a a a a
deriving ( Eq, Ord, Show )
```

The four parameters of this constructor specify the red, green and blue values of the color and additionally a fourth argument, which denotes the opaqueness of the color. The values are usually specified by floating numbers of type **GLfloat**. Values for number attributes are between 0 and 1.

You may wonder, why there is a special type GLfloat for numbers in HOpenGL. The reason is that OpenGL is defined in a way that it is as independent from concrete types in any implementation as possible. However you do not have to worry too much about this type. You can use ordinary float literals for numbers of type GLfloat. Haskells overloading mechanism ensures that these literals can create GLfloat numbers.

Example:

This program opens a window with a red background.

```
BackgroundColor.hs -
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
з
    main = do
4
      getArgsAndInitialize
\mathbf{5}
      createAWindow "red"
6
      mainLoop
7
8
    createAWindow windowName = do
9
      createWindow windowName
10
      displayCallback $= display
11
12
    display = do
13
      clearColor $= Color4 1 0 0 1
14
      clear [ColorBuffer]
15
```

Committing Complete Drawing

Whenever in a display function a sequence of monadic statements is defined, a final call to the function flush should be made. Only such a call will ensure that the statements are completely committed to the device, on which is drawn.

2.2.2 Primitive Shapes

So most preperatory things we know by now. We can start drawing onto the screen. Astonishingly in OpenGL there is only very limited number of shapes for drawing. Just points, simple lines and polygons. No curves or more complicated objects. Everything needs to be performed with these primitive drawing functions. The main function used for drawing something is **renderPrimitive**. The first argument of this functions specifies what kind of primitive is to be drawn. There are the following primitives defined in OpenGL:

```
data PrimitiveMode =
1
          Points
2
          Lines
3
          LineLoop
4
          LineStrip
\mathbf{5}
          Triangles
6
          TriangleStrip
7
          TriangleFan
8
          Ouads
9
          QuadStrip
10
          Polygon
11
        deriving ( Eq, Ord, Show )
12
```

The second argument defines the points which specify the primitives. These points are so called vertexes. Vertexes are actually monadic functions which constitute a point. If you want to define a point in a 3-dimensional universe with the coordinates x, y, z then you can use the following expression in HOpenGL:

```
vertex (Vertex3 x y z)
```

or, if you prefer the use of the standard prelude operator \$:

```
vertex$Vertex3 x y z
```

Points

We have seen in the introductory example that we can draw points. We can simply define a vertex and use this in the function renderPrimitiv.

Example:

This program draws one single yellow point on a black screen.

```
SinglePoints.hs

import Graphics.UI.GLUT

import Graphics.Rendering.OpenGL

main = do

getArgsAndInitialize

createAWindow "points"
```

```
mainLoop
7
8
    createAWindow windowName = do
9
      createWindow windowName
10
      displayCallback $= display
11
12
    display = do
13
      clear [ColorBuffer]
14
      currentColor $= Color4 1 1 0 1
15
      renderPrimitive Points
16
         (vertex (Vertex3 (0.1::GLfloat) 0.5 0))
17
      flush
18
```

If you do not like parantheses then you can of course use the operator \$ from the prelude and rewrite the line: renderPrimitive Points\$vertex\$Vertex3 (0.1::GLfloat) 0.5 0

Unfortunately Haskell needs sometimes a little bit of help for overloaded type classes. Therefore you find the type annotation (0.1::GLfloat) on one of the float literals. In larger applications Haskell can usually infer this information from the context. Just in smaller applications you will sometimes need to help Haskell's type checker a bit.

The second argument of **renderPrimitive** is a sequence of monadic statements. So, if you want more than one point to be drawn, you can define these in a nested *do statement*

Example:

In this program we use a nested *do statement* to define more points.

```
MorePoints.hs
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
3
   main = do
4
      getArgsAndInitialize
5
      createAWindow "more points"
6
      mainLoop
7
8
    createAWindow windowName = do
9
      createWindow windowName
10
      displayCallback $= display
11
12
    display = do
13
      clear [ColorBuffer]
14
      currentColor $= Color4 1 1 0 1
15
      renderPrimitive Points $
16
       do
17
          vertex (Vertex3 (0.1::GLfloat) 0.6 0)
18
          vertex (Vertex3 (0.1::GLfloat) 0.1 0)
19
      flush
20
```

If you want to think of points mainly as triples then you can convert a list of points into a sequence of monadic statements by first maping every triple into a vertex, e.g. by: map $(\langle x, y, z \rangle - vertex Vertex 3 x y z)$

and then combining the sequence of monadic statements into one monadic statement. Therefore you can use the standard function for monads: sequence_. The standard function mapM_ is simply the composition of map and sequence_, such that a list of triples can be converted to a monadic vertex statement by:

 $mapM_{(x,y,z)} \rightarrow vertex Vertex3 x y z)$ which is the technique used in the introductory example.

Example:

Thus we can rewrite a points example in the following way: points are defined as a list of triples. Furthermore we define some useful auxilliary functions:

```
EvenMorePoints.hs
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
3
    main = do
4
5
      getArgsAndInitialize
      createAWindow "more points"
6
      mainLoop
7
8
    createAWindow windowName = do
9
      createWindow windowName
10
      displayCallback $= display
11
12
    display = do
13
      clear [ColorBuffer]
14
      currentColor $= Color4 1 1 0 1
15
      let points = [(0.1, 0.6, 0::GLfloat)]
16
                      (0.2, 0.8, 0)
17
                      (0.3, 0.1, 0)
18
                      ,(0,0,0)
19
                      (0.4, -0.8, 0)
20
                      , (-0.2, -0.8, 0)
^{21}
                      1
22
      renderPoints points
23
      flush
^{24}
25
    makeVertexes = mapM (\langle x, y, z \rangle)->vertex$Vertex3 x y z)
26
27
    renderPoints = renderAs Points
28
29
    renderAs figure ps = renderPrimitive figure$makeVertexes ps
30
```

Some useful functions

In the following we want to explore all the other different shapes which can be rendered by OpenGL. All shapes are defined in terms of vertexes which you can think of as points. We have allready seen how to define vertexes and how to open a window and such things. We provide a simple module, which will be used in the consecutive examples. Some useful functions are defined in this module.

```
PointsForRendering.hs ______
1 module PointsForRendering where
2 import Graphics.UI.GLUT
3 import Graphics.Rendering.OpenGL
```

A first function will open a window und use a given display function for the window graphics:

```
PointsForRendering.hs ______

renderInWindow displayFunction = do

(progName,_) <- getArgsAndInitialize

createWindow progName

displayCallback $= displayFunction

mainLoop
```

The next function creates for a list of points, which are expressed as triples, and a basic shape a display function which renders the desired shape.

```
PointsForRendering.hs

displayPoints points primitiveShape = do

renderAs primitiveShape points

flush

renderAs figure ps = renderPrimitive figure$makeVertexes ps

makeVertexes = mapM_ (\(x,y,z)->vertex$Vertex3 x y z)
```

Eventually we define a list of points as example and provide a function for easy use of these points:

```
_ PointsForRendering.hs _
16
    mainFor primitiveShape
     = renderInWindow (displayMyPoints primitiveShape)
17
18
    displayMyPoints primitiveShape = do
19
      clear [ColorBuffer]
^{20}
      currentColor $= Color4 1 1 0 1
^{21}
      displayPoints myPoints primitiveShape
^{22}
23
    myPoints
^{24}
     = [(0.2,-0.4,0::GLfloat)
^{25}
        , (0.46, -0.26, 0)
^{26}
        ,(0.6,0,0)
27
        ,(0.6,0.2,0)
^{28}
        ,(0.46,0.46,0)
^{29}
        (0.2, 0.6, 0)
30
        ,(0.0,0.6,0)
31
```

Example:

We can now render the example points in a oneliner:

```
RenderPoints.hs

import PointsForRendering

import Graphics.Rendering.OpenGL

main = mainFor Points
```

Lines

The next basic thing to do with vertexes is to connect them, i.e. consider them as starting and end point of a line. There are three ways to connect points with lines in OpenGL.

Singleton Lines The most natural way is to take pairs of points and draw lines between these. This is done in the primitive mode Lines. In order that this works properly an even number of vertexes needs to be supplied to the function renderPrimitive.

Example:

Connecting our example points by lines. Pairs of points define singleton lines.

The resulting window can be found in figure 2.1.

Line Loops The next way to connect points with lines you probably can imagine is to make a closed figure. The end point of a line is the starting point of the next line and the last point is connected with the first, such that a closed loop of lines is created.

Example: Now we make a loop of lines with our example points.

```
RenderLineLoop.hs

import PointsForRendering

import Graphics.Rendering.OpenGL

main = mainFor LineLoop
```

The resulting window can be found in figure 2.2.

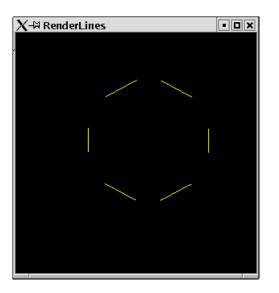


Figure 2.1: Lines between points.

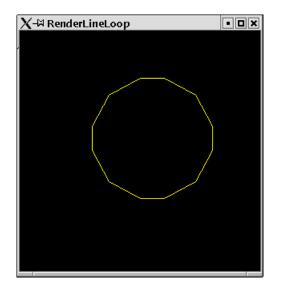
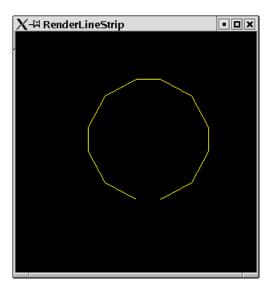


Figure 2.2: A loop of lines.

Line Strip A strip of lines is very close to a loop of lines. The only thing missing is the last line which connects the last point with the first one again.

Example:

Now we make a strip of lines with our example points.



The resulting window can be found in figure 2.3.

Figure 2.3: A strip in terms of lines.

Triangles

The next basic shape which can be rendered by OpenGL are triangles. Triples of points are taken and triangles are drawn with these. As for lines there are three flavours of triangles.

Triangle The most natural way of drawing triangles is to take triples and draw triangles. In order to work for triangles, the number of points provided needs to be a multiple of 3.

Example:

Our example vertexes define 12 points such that we get 4 triangles

```
RenderTriangles.hs -

import PointsForRendering

import Graphics.Rendering.OpenGL

main = mainFor Triangles
```

The resulting window can be found in figure 2.4.

Triangle Strips A triangle strip makes a sequence of triangles where the next triangle uses two points of its predecessor and one new point.

Example:

For our 12 points a triangle strip will create 10 triangles.

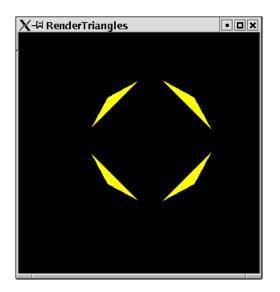


Figure 2.4: Triangles.

	RenderTriangleStrip.hs
1	import PointsForRendering
2	import Graphics.Rendering.OpenGL
3	
4	main = mainFor TriangleStrip

The resulting window can be found in figure 2.5.

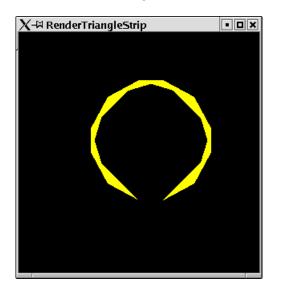


Figure 2.5: A triangle strip.

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TriangleFan A fan has one starting point for all triangles. Triangles are always drawn starting from the first point.

Example:

Our example points as a fan. 10 triangles are rendered.

The resulting window can be found in figure 2.6.

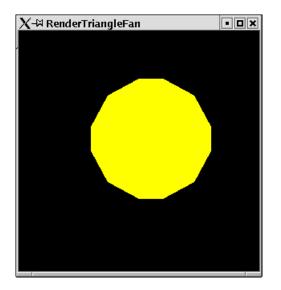


Figure 2.6: A triangle strip.

Quads

Lines connected two points, triangles three points, now we will connect four points. This is calles a *quad*. There are two flavours of quads.

Singleton Quads The primitive mode **Quads** takes quadruples of points and connects them in order to render a filled figure.

Example:

For our 12 example points OpenGL renders 3 quads $% \left({{{\rm{A}}} \right)$

```
RenderQuads.hs

import PointsForRendering

import Graphics.Rendering.OpenGL

main = mainFor Quads
```

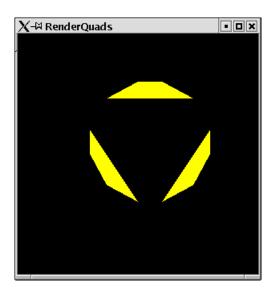


Figure 2.7: Quads.

The resulting window can be found in figure 2.7.

In a three dimensional world quads are unlike triangles not necessarily plane areas.

QuadStrips For a strip of quads OpenGL uses two points of the preceeding quads for the next quad. The number n of vertexes therefore needs to be of the form: n = 4 + 2 * m.

1	import PointsForRendering
2	import PointsForRendering import Graphics.Rendering.OpenGL
3	
4	main = mainFor <u>QuadStrip</u>

The resulting window can be found in figure $2.8.^3$

Polygons

We connected two, three and for points. Eventually there is a shape that connects an arbitrary number of points. This is generally called a polygon. There are some restrictions for polygons:

- no convex corners are allowed.
- lines may not cross each other.

³Which somehow does not look like the expected?

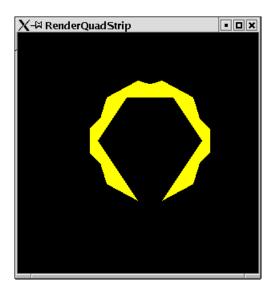


Figure 2.8: QuadStrips.

• polygons need to be planar.

Example:

Eventually our vertexes are used to define a polygon.

```
RenderPolygon.hs ______

import PointsForRendering

import Graphics.Rendering.OpenGL

main = mainFor Polygon
```

In this case the resulting window looks like the triangle fan we have seen before.

If you want to render polygons which hurt some of the restrictions above, you need to represent them by a set of smaller polygons. Since this is a tedious task to be done manually there is a library available, which does this for you: the GLU tessellation.

2.2.3 Curves, Circles and so on

In the last sections you have seen all primitive shapes, which can be rendered by OpenGL. Everything else needs to be constructed in term of these primitives. Especially you might wonder where curves and circles are. The bad news is: you have to do these by yourself.

Circles

With a bit mathematics you probably have allready guessed how to do curves and especially circles. You need to approximate them with a large number of lines. If the lines get very small we eventually see a curve. Let us try this with circles. We write a module which gives us some utility functions for rendering circles.

_ Circle.hs .

```
1 module Circle where
2 import PointsForRendering
```

```
3 import Graphics.Rendering.OpenGL
```

The crucial function calculates a list of points which are all on the circle. You need a bit of basic geometrical knowledge for this. The coordinates of the points on a circle can be determined by $\sin(\alpha)$ and $\cos(\alpha)$ where α is between 0 and 2π .

Thus we can easily calculate the coordinates of an arbitrary number of points on a circle:

```
Circle.hs

circlePoints radius number

= [let alpha = twoPi * i /number

in (radius*(sin (alpha)) ,radius * (cos (alpha)),0)

|i <- [1,2..number]]

where

twoPi = 2*pi
```

If we take a large anough number then we will eventually get a circle:

```
10 Circle radius = circlePoints radius 100
```

The following function can be used to render the circle figures:

```
Circle.hs

renderCircleApprox r n

= displayPoints (circlePoints r n) LineLoop

renderCircle r = displayPoints (circle r) LineLoop

fillCircle r = displayPoints (circle r) Polygon
```

Example:

First we test what kind of shape we get for small approximation numbers.

The resulting graphic can be seen in figure 2.9.

Example:

Now we can test, if the resulting circle is, what we expected.

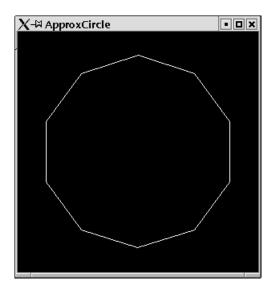


Figure 2.9: 10 points on a circle.

```
TestCircle.hs -

import PointsForRendering

import Circle

import Graphics.Rendering.OpenGL

main = renderInWindow $ do

clear [ColorBuffer]

renderCircle 0.8
```

The resulting graphic can be seen in figure 2.10.

Example:

And eventually have a look at the filled circle.

```
_ FillCircle.hs _
   import PointsForRendering
1
   import Circle
^{2}
3
   import Graphics.Rendering.OpenGL
4
\mathbf{5}
   main
6
    = renderInWindow $ do
7
         clear [ColorBuffer]
8
         fillCircle 0.8
9
```

The resulting graphic can be seen in figure 2.11.

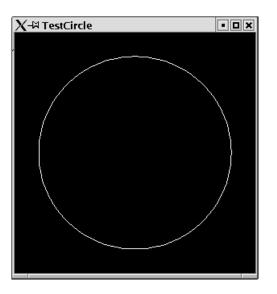


Figure 2.10: Rendering a full circle.

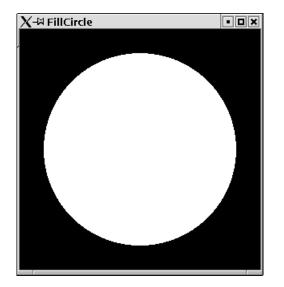


Figure 2.11: A filled circle.

Rings

Now, where you know how to do circles, you can equally as easy define functions for rendering rings. A ring has an inner and an outer circle and fills the space between these. So we can approximate these two rings and render quads between them.

_____ Ring.hs _

```
1 module Ring where
2
3 import PointsForRendering
4 import Circle
```

2-19

import Graphics.Rendering.OpenGL

We can simply define the points of the inner and outer ring and merge these. The resulting list of points can then be rendered as a QuadStrip. Since there is no primitive mode for quad loops, we need to append the first two points as the last points again:

```
Ring.hs

ringPoints innerRadius outerRadius

concat$map (\(x,y)->[x,y]) (points++[p])

where

innerPoints = circle innerRadius

outerPoints = circle outerRadius

points@(p:_) = zip innerPoints outerPoints
```

Eventually we provide a small function for rendering ring shapes.

```
Ring.hs ______

12 ring innerRadius outerRadius _____

13 = displayPoints (ringPoints innerRadius outerRadius) QuadStrip
```

Example:

We can test the ring functions:

```
TestRing.hs

import PointsForRendering

import Ring

import Graphics.Rendering.OpenGL

main = renderInWindow $ do

clear [ColorBuffer]

ring 0.7 0.9
```

The resulting graphic can be seen in figure 2.12.

2.2.4 Attributes of primitives

There are some more attributes that can be set for primitive shapes (besides the color, which we have allready set).

Point Size

You could argue that there is no need for single points. A point can be modelled by a circle that has a small radius (or in the third dimension a sphere). However, there is something like a point in OpenGL and you can set its size. This size value for points does not refer to a radius in the coordinate system but is measured in terms of screen pixels. The default value is, one pixel per point.

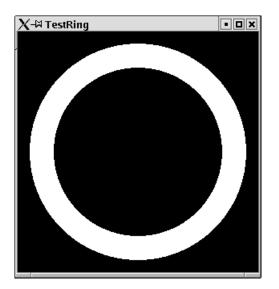


Figure 2.12: A simple ring shape.

Example:

We set the point size to 10 pixels:

```
PointSize.hs

import Graphics.Rendering.OpenGL

import PointsForRendering

main = renderInWindow display

display = do

pointSize $= 10

displayMyPoints Points
```

The resulting graphic can be seen in figure 2.13.

Line Attributes

As for points, there are also further attributes for lines. First of all there is a line width. As for the point size, this is measured in screen pixels. Furthermore, you can set some line stipple: this is the pattern of the line, dashes etc. For the line stipple there is a state variable of type: Maybe (GLint, GLushort). The second argument of the value pair denotes the kind of stipple. For every short value there is one stipple. The short value has 16 bits. Every bit stands for a pixel. If for the corresponding short number the bit is set, then the pixel will be drawn, otherwise not. This means that for the short number 0 you will not see anything of your line, and for the value 65535 you will see a solid line.

The integer number of the value pair denotes a factor for the chosen stipple. For some positiv integer n every bit of the short number stands for n bits.

Example:

Setting the width of lines and a stipple:

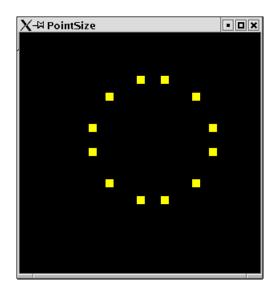


Figure 2.13: Points of a large size.

```
_ LineAttributes.hs .
    import Graphics.Rendering.OpenGL
1
    import Graphics.UI.GLUT as GLUT
2
    import PointsForRendering
3
4
    main = renderInWindow display
5
6
    display = do
7
      clearColor $= Color4 1 1 1 1
8
      clear [ColorBuffer]
9
      lineStipple $= Just (1,255)
10
      currentColor $= Color4 0 0 0 1
11
      lineWidth $= 10
12
      displayPoints squarePoints LineLoop
13
      flush
14
15
    squarePoints
16
     = [(-0.7, -0.7, 0), (0.7, -0.7, 0), (0.7, 0.7, 0), (-0.7, 0.7, 0)]
17
```

The resulting graphic can be seen in figure 2.14.

Colors

You might have wondered, why the function renderPrimitive takes monadic statements as argument and not simply a list of vertexes? This means we could pass any monadic statement to the function renderPrimitive, not only statements that define vertexes by the call of the function vertex. There are some statements, which are allowed in the statements passed to renderPrimitive. One of these is setting the current color before every call of vertex

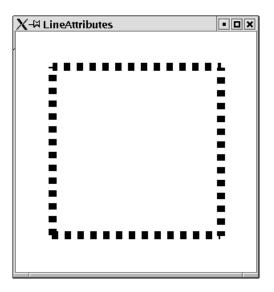


Figure 2.14: Thick stippled lines.

to a new value. When finally rendering the primitive, OpenGL takes these color values into acount.

Example:

We define a triangle. Before the three vertexes of the triangle are defined, the current color is set to a new value.

```
- PolyColor.hs
    import Graphics.Rendering.OpenGL
1
    import Graphics.UI.GLUT as GLUT
2
    import PointsForRendering
3
4
    colorTriangle = do
\mathbf{5}
      currentColor $= Color4 1 0 0 1
6
      vertex$Vertex3 (-0.5) (-0.5) (0::GLfloat)
7
      currentColor $= Color4 0 1 0 1
8
      vertex$Vertex3 (0.5) (-0.5) (0::GLfloat)
9
      currentColor $= Color4 0 0 1 1
10
      vertex$Vertex3 (-0.5) (0.5) (0::GLfloat)
11
12
    main = renderInWindow display
^{13}
14
    display = do
15
      clearColor $= Color4 1 1 1 1
16
      clear [ColorBuffer]
17
      renderPrimitive Triangles colorTriangle
18
      flush
19
```

The resulting window can be found in figure 2.15.

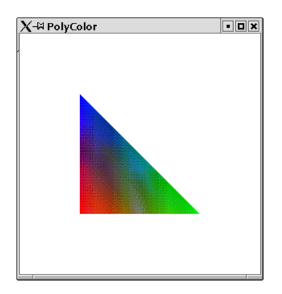


Figure 2.15: A triangle with different vertex colors

2.2.5**Tessellation**

Rendering of polygons is very limited. We cannot render polygons for crossing lines, or convex corners. Such polygons need to be expressed by a set of simpler polygons. In the module Graphics.Rendering.OpenGL.GLU.Tessellation there are a number of functions, which calculate a set of simpler polygons. For the time being, we will not go into detail, but give one single example, of how to use this library.

Example:

We want to render stars. These are shapes with convex corners.

		Star.ns
1	module	Star where
2	import	Graphics.Rendering.OpenGL
3	import	Graphics.UI.GLUT as GLUT
4	import	Data.Either
5	import	Circle
6	import	List

Ctow ha

We can easily calculate the points on the star rays. They are all on one circle. We can use our function for defining circle points and get a list of points. For rendering the star, we take first the points with odd index followed by the points with even index.

_ Star.hs _

```
starPoints radius rays
7
     = map (\(_,(x,y,z))->Vertex3 x y z)(os++es)
8
      where
9
       (os, es) = partition (((i, _)-> odd i))
10
                    $zip [1,2..]
11
                    $circlePoints radius rays
12
```

For tesselation we need to create a ComplexPolygon, which has a list of ComplexContour. A ComplexContour contains a list of AnnotatedVertexes. The annotation can be used for color or similar information. We do not make use of this annotation and simple annotate every vertex with 0.

```
Star.hs

complexPolygon points

= ComplexPolygon

[ComplexContour $map (\v->AnnotatedVertex v 0) points]
```

The function **tesselate** creates a list of simple polygons. It needs some control information, which we do not explain here.

```
Star.hs

star radius rays= do

startess

- tessellate

TessWindingPositive 0 (Normal3 0 0 0) noOpCombiner

$complexPolygon (starPoints radius rays)

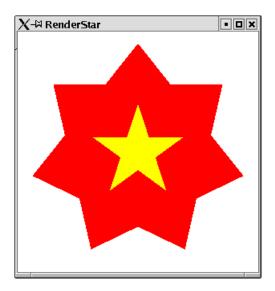
drawSimplePolygon startess
```

The resulting simple polygons can be rendered with the function renderPrimitive.

```
_ Star.hs _
    drawSimplePolygon (SimplePolygon primitiveParts) =
22
      mapM_ renderPrimitiveParts primitiveParts
23
^{24}
    renderPrimitiveParts (Primitive primitiveMode vertices) =
^{25}
      renderPrimitive primitiveMode
26
        $mapM_ (vertex . stripAnnotation) vertices
27
28
    stripAnnotation (AnnotatedVertex plainVertex _) = plainVertex
29
30
   noOpCombiner _newVertex _weightedProperties = 0.0 ::GLfloat
31
```

Now we can test our stars. We render two stars, one with 7 and one with 5 rays.

```
_ RenderStar.hs _
   import PointsForRendering
1
   import Graphics.Rendering.OpenGL
2
   import Graphics.UI.GLUT as GLUT
3
   import Star
4
5
   main = renderInWindow$do
6
     clearColor $= Color4 1 1 1 1
7
     clear [ColorBuffer]
8
9
     currentColor $= Color4 1 0 0 1
10
     star 0.9 7
11
12
     currentColor $= Color4 1 1 0 1
13
     star 0.4 5
14
```



The resulting window can be found in figure 2.16.

Figure 2.16: RenderStar

2.2.6 Cubes, Dodecahedrons and Teapots

The bad news was that just very basic shapes are provided by OpenGL for rendering. The good news is that the OpenGL library comes along with a library that contains a large number of shapes.

Example:

You probably need very often the shape of a teapot. Since this is so elementary a library function is provided for this.

Tea.hs

```
import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
^{2}
3
    import PointsForRendering
4
\mathbf{5}
    main = renderInWindow display
6
7
    display = do
8
      clear [ColorBuffer]
9
      renderObject Solid$ Teapot 0.6
10
      flush
11
```

The resulting graphic can be seen in figure 2.17.



Figure 2.17: A tea pot.

Chapter 3

Modelling Transformations

By now you know, how to define different shapes for rendering. You might wonder how to place shapes on special positions or how to scale or rotate your shapes. This is done by so called transformation matrixes. Before something is rendered by OpenGL a transformation operation is performed on it. Every point will get multiplied with the transformation matrix. The transformation matrix is part of the state. So in order to transform a shape in some way, first the transformation matrix has to be set and then the shapes are to be rendered. If not specified otherwise the transformation matrix is the identity operation, i.e. no transformation is performed. You can always reset the transformation matrix to the identity by the call of the monadic statement loadIdentity. Then the current matrix is discarded and no transformation is applied to the next rendering operations.

3.1 Translate

One transformation is to move a shape to another position. The according matrix is set by the statement translate. It has one argument: a vector of size three which denotes in which direction the following shapes are to be moved. Every vertex that will be rendered after a translate statement will be moved by the values of this vector.

Example:

The function **ring** we defined before only defined rings which have the center coordinates (0, 0, 0). If we want to place rings somewhere else then we need to apply a translate matrix.

```
_____ SomeRings.hs
```

```
import PointsForRendering
```

```
13 import Ring
```

12

```
14 import Graphics.Rendering.OpenGL
```

We define a function, which creates a ring at a given position. Therefore we first set the transformation to the translate transformation then define the ring and finally set the transformation matrix back to the identity:

We can test this by placing some ring in different colors on the screen.

```
_ SomeRings.hs
    main = do
18
      renderInWindow
                        someRings
19
20
    someRings = do
^{21}
      clearColor $= Color4 1 1 1 1
22
      clear [ColorBuffer]
^{23}
24
      loadIdentity
25
      currentColor $= Color4 1 0 0 1
26
      ringAt 0.5 0.3 0.1 0.12
27
28
      loadIdentity
29
      currentColor $= Color4 0 1 0 1
30
      ringAt (-0.5) 0.3 0.3 0.5
31
32
      loadIdentity
33
      currentColor $= Color4 0 0 1 1
34
      ringAt (-1) (-1) 0.7 0.75
35
36
      loadIdentity
37
      currentColor $= Color4 0 1 1 1
38
      ringAt 0.7 0.7 0.2 0.3
39
```

The resulting graphic can be seen in figure 3.1.

Note that if we did not reset the transformation back to the identity, we would get the composition of all transformations.

3.2 Rotate

Another transformation that can be performed is rotation. The rotate statement has two arguments. The first one specifies by which degree the following shapes are to be rotated counterclockwise. The second argument is a vector which specifies around which axis the shape is to be rotated.

Example:

In this example we apply the composition of two transformations. Squares are moved to some position and furthermore rotated around the z-axis.

We write a simple module for rendering filled rectangles:

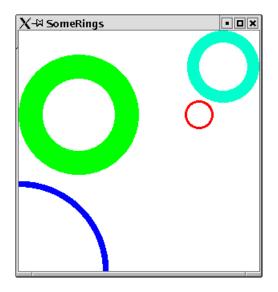


Figure 3.1: Rings translated to different positions.

```
Squares.hs
module Squares where
import Graphics.Rendering.OpenGL
import PointsForRendering
```

Here is a function for arbitrary rectangles:

```
____ Squares.hs __
```

```
5
6 myRect width height =
7 displayPoints [(w,h,0),(w,-h,0),(-w,-h,0),(-w,h,0)] Quads
8 where
9 w = width/2
10 h = height/2
```

A square is just a special case:

1

1

2

	-	•	-				
					Sa	uares.h	s
1	square	width	=	myRect			

Now we will transform squares.

_____ SomeSquares.hs __

```
import PointsForRendering
import Squares
```

```
3 import Graphics.Rendering.OpenGL
```

We define a function, which applies the rotate transformation to a square. It is rotated around the z-axis.

A further utility function moves some shape to a specified position. Note that this function resets the matrix again.

```
SomeSquares.hs
displayAt x y displayMe = do
translate$Vector3 x y (0::GLfloat)
displayMe
loadIdentity
```

Some squares are defined and rotated:

```
_ SomeSquares.hs _
    main = do
11
      renderInWindow
                        someSquares
12
13
    someSquares = do
14
      clearColor $= Color4 1 1 1 1
15
      clear [ColorBuffer]
16
17
      currentColor $= Color4 1 0 0 1
18
      displayAt 0.5 0.3$rotatedSquare
                                           15 0.12
19
20
      currentColor $= Color4 0 1 0 1
21
      displayAt (-0.5) 0.3$rotatedSquare 25 0.5
22
23
      currentColor $= Color4 0 0 1 1
^{24}
      displayAt (-1) (-1)$rotatedSquare 4 0.75
25
26
      currentColor $= Color4 0 1 1 1
27
      displayAt 0.7 0.7$rotatedSquare 40 0.3
28
```

The resulting graphic can be seen in figure 3.2.

3.3 Scaling

The third transformation enables you to scale shapes. This is not only useful for changing the size of some object but for stretching it in some direction. The transformation scale has three arguments, which represent the scaling factors in the three dimensional space.

Example:

We apply three transformations on the tea pot example. We rotate and translate it and finally we stretch it a bit by a scale transformation.

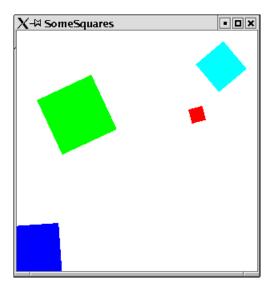


Figure 3.2: Squares translated and rotated.

```
Coffee.hs
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
3
    import PointsForRendering
4
    main = renderInWindow display
5
6
    display = do
7
      clear [ColorBuffer]
8
      scale 0.3 0.9 (0.3::GLfloat)
9
      translate$Vector3 (-0.3) 0.3 (0::GLfloat)
10
      rotate 30 $Vector3 0 1 (0::GLfloat)
11
      renderObject Solid$ Teapot 0.6
12
      loadIdentity
13
      flush
14
```

The resulting graphic can be seen in figure 3.3. As you see it looks now like a coffee pot.

Remember that the scale and the rotate transformation always refer to the origin (0,0,0) of your coordinates. Rotating an object, which is not situated at the origin will move it around the origin. Scaling an object which is not situated at the origin might deform the object in surprising ways.

3.4 Composition of Transformations

Since Haskell is a functional programming language let us think of transformations as functions. A transformation is a function that is applied to every vertex before it is rendered.



Figure 3.3: A coffee pot.

If you define two transformations for an object, e.g. a rotation and a translation, then you define a composition of these transformations.

The code:

```
1 rotatedSquareAt width alpha x y z = do
2 translate$Vector3 x w y
3 rotate alpha $Vector3 0 0 (1::GLfloat)
4 square width
```

defines a composition of a translate und a rotate transformation, which is applied to a square figure. A sequence of transformation statements is composed to a single transformation in the same way as the standard function composition operator (.) composes functions: $(f \cdot g) = f(g(x))$. The compositional function $(f \cdot g)$ is the same as first applying function g and then applying f. For transformations in HOpenGL this means that for a sequence of transformations

```
1 translate$Vector3 x w y
2 rotate alpha $Vector3 0 0 (1::GLfloat)
```

first the points are rotated and then they are translated.

The order in which transformations are performed is of course not arbitrary. A rotation after a translation is different to a translation after a rotation.

Example:

This example illustrates the different compositions of rotation and translation.

Compose.hs

```
<sup>1</sup> import PointsForRendering
```

```
<sup>2</sup> import Squares
```

```
import Graphics.Rendering.OpenGL
3
4
\mathbf{5}
    displayAt x y displayMe = do
      displayMe
6
      loadIdentity
7
8
    main = do
9
      renderInWindow someSquares
10
11
    someSquares = do
^{12}
      clearColor $= Color4 1 1 1 1
13
      clear [ColorBuffer]
14
```

A black square at the origin:

```
Compose.hs ______

currentColor $= Color4 0 0 0 1

square 0.5

loadIdentity
```

A blue square translated:

```
Compose.hs ______

currentColor $= Color4 0 0 1 1

translate$Vector3 0.5 0.5 (0::GLfloat)

square 0.5

loadIdentity
```

A light blue square that is rotated:

```
Compose.hs -
currentColor $= Color4 0 1 1 1
rotate 35 $Vector3 0 0 (1::GLfloat)
square 0.5
loadIdentity
```

A red square that is first rotated and then translated:

```
Compose.hs

currentColor $= Color4 1 0 0 1

translate$Vector3 0.5 0.5 (0::GLfloat)

rotate 35 $Vector3 0 0 (1::GLfloat)

square 0.5

loadIdentity
```

A yellow square that is first translated and then rotated:

```
Compose.hs

31 currentColor $= Color4 1 1 0 1

32 rotate 35 $Vector3 0 0 (1::GLfloat)

33 translate$Vector3 0.5 0.5 (0::GLfloat)

34 square 0.5

35 loadIdentity
```

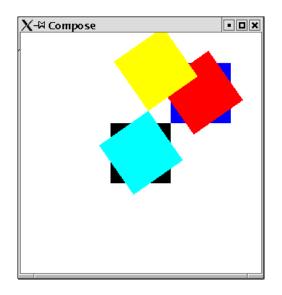


Figure 3.4: Different compositions of translation and rotation.

The resulting window can be found in figure 3.4.

Since the scale und the rotate transformation refer both to the origin and the translate transformation can move objects away from the origin it is a good policy to create objects at the origin, then rotate and scale it and finally translate it to its final position. Therefore predefined shapes in the library are usually positioned at the origin, as e.g. the tea pot.

3.5 Defining your own transformation

The three ready to use transformations rotation, scaling and translation or their composition might not suffice for your needs. Then you can define your own transformations. Technically a transformation in OpenGL is represented as a matrix. Every vertex gets multiplied by the transformation matrix before it is rendered. In order to define a transformation, we will need to construct such a matrix.

Internally every vertex in OpenGL is not represented by 3 coordinates (x, y, z) but by four coordinates (x, y, z, w). The x, y, z values are devided by w. Usually the value of w is 1.0.

Thus for a transformation matrix you need a matrix of four rows and four columns. Remember that a matrix is multiplied with a vector in the following way:

1	x_{11}	x_{12}	x_{13}	x_{14}	$\setminus I$	x)	$(x_{11} * x + x_{12} * y + x_{13} * z + x_{14} * w)$
	x_{21}			x_{24}		y	_	$x_{21} * x + x_{22} * y + x_{23} * z + x_{24} * w$
	x_{31}	x_{32}	x_{33}	x_{34}		z		$x_{31} * x + x_{32} * y + x_{33} * z + x_{34} * w$
(x_{41}	x_{42}	x_{43}	x_{44} ,	/ \	$\langle w \rangle$)	$(x_{41} * x + x_{42} * y + x_{43} * z + x_{44} * w)$

OpenGL provides a function for creation of a transformation matrix out of a list: matrix. It takes as first argument a parameter, which specifies in which order the matrix elements appear in the list: RowMajor for row wise and ColumMajor for column wise appearance. The function multMatrix allows to multiply your newly created transformation matrix to the current transformation context.

3.5.1 Shear

We can now define our own transformations. We can define the transformation *shear*. Mathematical textbooks define *shear* in the following way:

A transformation in which all points along a given line L remain fixed while other points are shifted parallel to L by a distance proportional to their perpendicular distance from L. Shearing a plane figure does not change its area. *Eric Weissteins's world of mathematics* (http://mathworld.wolfram.com/Shear.html)

We define a *shear* transformation, which leaves y and z coordinates unchanged, and adds to the x coordinate some value depending on the value of y. For some f we need the following transformation matrix:

/ 1	f	0	0 \	(x)		(x+f*y)	١
0	1	0	0	$ \left(\begin{array}{c} x \\ y \\ z \end{array}\right) $	_	y	
0	0	1	0	z	=	z	
$\int 0$	0	0	$\left(\begin{array}{c}0\\0\\0\\1\end{array}\right)$	$\left(\begin{array}{c} w \end{array} \right)$		$\langle w \rangle$	/

As you can see, this is almost the identity. We can define this in HOpenGL:

```
MyTransformations.hs _____
    module MyTransformations where
1
    import Graphics.Rendering.OpenGL
2
    import Graphics.UI.GLUT as GLUT
3
4
    shear f = do
\mathbf{5}
      m <- (newMatrix RowMajor [1,f,0,0</pre>
6
                                      ,0,1,0,0
7
                                      ,0,0,1,0
8
                                      ,0,0,0,1])
9
      multMatrix (m:: GLmatrix GLfloat)
10
```

Let us test our new transformation:

```
_ TestShear.hs _
    import PointsForRendering
1
    import Circle
2
    import Squares
3
    import MyTransformations
4
\mathbf{5}
    import Graphics.Rendering.OpenGL
6
    import Graphics.UI.GLUT as GLUT
7
8
    main = renderInWindow$do
9
     loadIdentity
10
     clearColor $= Color4 1 1 1 1
11
     clear [ColorBuffer]
12
     translate$Vector3 0.5 0.5 (0::GLfloat)
13
     shear 0.5
14
     currentColor $= Color4 0 0 1 1
15
```

```
16 fillCircle 0.5
17
18 loadIdentity
19 translate$Vector3 (-0.5) (-0.5) (0::GLfloat)
20 shear 0.5
21 currentColor $= Color4 1 0 0 1
22 square 0.5
```

The resulting window can be found in figure 3.5.

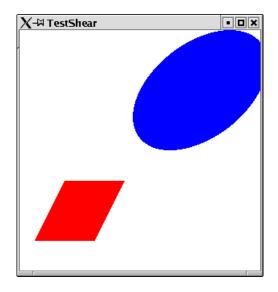


Figure 3.5: Applying shear to some shapes.

3.6 Some Word of Warning

You might get strange effects when you forget to reset the transformation matrix. This might not only effect further rendering statements but also applies to the redisplay of your window. The display function you specified for your window will be called whenever the window needs to be displayed. However this does not automatically reset the transformation matrix to the identity matrix. This results in the effect that every redisplay of your window changes its contents.

Example:

In this example a ring is displayed. Each time the display function is called the contents of the ring moves a bit. Compile the program and hide the resulting window behind some other window. You will observe how the ring moves within the window, until it is no longer displayed.

ForgottenReset.hs

- import Graphics.UI.GLUT
- ² import Graphics.Rendering.OpenGL

3

1

```
import PointsForRendering
4
5
    import Ring
    import Squares
6
7
    main = renderInWindow display
8
9
    display = do
10
      clear [ColorBuffer]
11
      translate$Vector3 (-0.1) 0.1 (0::GLfloat)
12
      ring 0.2 0.4
13
      flush
14
```

As a matter of fact this effect may not only occur with transformations, but every state changing statement. If you set the color as last statement in your display function to some value then this will be the current color in the next call of the display function. Thus it is better to ensure that the display function leaves a clean state, i.e. the state it espects to find, when it is called, or even better let the display functions not rely on any previously set states.

3.7 Local transformations

Often you will have the situation, that you are in a context of some transformations. Maybe for certain parts of you shape you want to add some further transformation but for other parts return to the outer transformation context. In such situations you cannot use the statement loadIdentity since this will not only delete the transformations you wanted to be applied to your local part of the the complete shape but the whole transformation context.

HOpenGL provides a function which allows to add some more transformations to some local parts of your shape. This function is called **preservingMatrixs** which refers to the fact that transformations are technically implemented as matrixes. **preservingMatrix** has one argument, which is a monadic statement. The application of **preservingMatrix** is a monadic statement:

```
preservingMatrix :: IO a -> IO a
```

Every transformation done within this monadic statement will not be done only locally. It does not effect the statements which follow after the application of preservingMatrix.

Example:

To demonstrate the use of preservingMatrix we provide a module, which is able to render a side of the famous Rubik's Cube. Such a side consists of 9 squares which are of some color and which have a black frame. We can render such a shape, by rendering the single framed squares at the origin and then move them to their position. This movement is done within a preservingMatrix application.

```
RubikFace.hs -
module RubikFace where
mport Graphics.UI.GLUT
mport Graphics.Rendering.OpenGL
mport Squares
mport Squares
import PointsForRendering
```

Doing a frame involves the four sides of a frame. Each side is created at the origin and then moved to its final position:

```
_ RubikFace.hs _
    frame width height border = do
7
      let bh = border/2
8
      let wh = width/2-bh
9
      let hh = height/2-bh
10
11
      preservingMatrix $ do
12
        translate $Vector3 0 hh (0::GLfloat)
13
        myRect width border
14
      preservingMatrix $ do
15
        translate $Vector3 0 (-hh) (0::GLfloat)
16
        myRect width border
17
      preservingMatrix $ do
^{18}
        translate $Vector3 (-wh) 0 (0::GLfloat)
19
        myRect border height
20
      preservingMatrix $ do
21
        translate $Vector3 wh 0 (0::GLfloat)
^{22}
        myRect border height
23
```

Each of the nine fields is rendered by drawing its frame and its colored square:

```
RubikFace.hs —

originField width color = do

let frameWidth = width/10

currentColor $= Color4 0 0 0 1

frame width width frameWidth

let sc = 18/20::GLfloat

currentColor $= color

square (width-frameWidth)
```

Eventually the side of Rubik's Cube can be drawn

```
_ RubikFace.hs
    renderArea :: GLfloat -> [[Color4 GLfloat]] -> IO ()
31
   renderArea width css
32
    = do
33
        let cs = concat css
^{34}
            cps = zip cs $ areaFields width
35
        mapM_ (\(c,f)-> f(originField width c)) cps
36
37
   areaFields width =
38
```

```
[makeSquare x y |x<-[1,0,-1],y<-[1,0,-1]]
39
40
        where
          makeSquare xn yn = f \rightarrow \text{preservingMatrix } do
41
            let
42
              x = xn*width
^{43}
              y = yn*width
44
            translate $Vector3 x y 0
45
            f
46
47
             = Color4 1 0 0 (1::GLfloat)
    red
^{48}
             = Color4 0 1 0 (1::GLfloat)
    green
49
            = Color4 0 0 1 (1::GLfloat)
    blue
50
    yellow = Color4 1 1 0 (1::GLfloat)
51
    white
             = Color4 1 1 1 (1::GLfloat)
52
   black
             = Color4 0 0 0 (1::GLfloat)
53
```

The following module tests the rendering. Two sides are rendered. Further transformations are applied to them.

```
_ RenderRubikFace.hs _
    import PointsForRendering
1
    import Graphics.Rendering.OpenGL
2
3
    import PointsForRendering
4
    import RubikFace
5
6
    _FIELD_WIDTH :: GLfloat
7
    _FIELD_WIDTH = 1/5
8
9
    main = renderInWindow faces
10
11
    faces = do
12
      clearColor $= white
13
      clear [ColorBuffer]
^{14}
15
      loadIdentity
16
      translate $Vector3 (-0.6) 0.4 (0::GLfloat)
17
      renderArea _FIELD_WIDTH r1
18
^{19}
      loadIdentity
20
      translate $Vector3 (0.1) (-0.3) (0::GLfloat)
21
      rotate 290 $ Vector3 0 0 (1::GLfloat)
22
      scale 1.5 1.5 (1::GLfloat)
23
      renderArea _FIELD_WIDTH r1
^{24}
25
    r1=[[red,blue,yellow],[white,green,red],[green,yellow,blue]]
26
```

The resulting window can be found in figure 3.6.

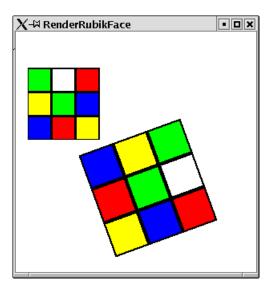


Figure 3.6: a Side of Rubik's Cube with further transformations applied to it.

Chapter 4

Projection

4.1 The Function Reshape

Up to now we always relied on the default values for most attributes which are concerned with projection. From where do we look at the scenery? Which coordinates are displayed to what extend on the screen. Such attributes can be set in the reshape callback function. This function gets the window size as argument and specifies which coordinates are to be seen on the screen. At first glance the name seems to be a bit misleading, since it evokes the image that it is just called, when someone resizes the window. The first time the reshape function is called is at the opening of the window.

The reshape function might be empty. This is modelled by the Haskell data type Maybe.

Example:

We define the first reshape function for a window. It is the identity function, which does not specify anything, how to render the picture.

Reshape1.hs

```
import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
з
    import PointsForRendering
4
5
    main = do
6
      (progName,_) <- getArgsAndInitialize</pre>
7
      createWindow progName
8
      displayCallback $= display
9
      reshapeCallback $= Just reshape
10
      mainLoop
11
12
    display = do
13
     clear [ColorBuffer]
14
     displayPoints points Quads
15
       where
16
        points
17
         = [(0.5, 0.5, 0)]
18
```

19	,(-0.5,0.5,0)
20	,(-0.5,-0.5,0)
21	,(0.5,-0.5,0)]
22	
23	reshape s = return ()

Run this example. You will see a white square in the middle of a black screen. Now resize the window. You will notice that the size of the square will not change. If you make the window smaller parts of the picture are not displayed, if you enlarge the window parts of the window contain no image (which means it might be some arbitrary image). Figure 4.1 shows how the window looks after enlarging it a bit.

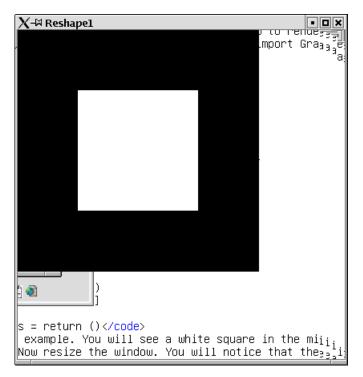


Figure 4.1: Enlarging a window with the empty reshape function.

Viewport: The Visible Part of Screen 4.2

Usually you want to define in the reshape function, which parts of the window pane are to be used for rendering the picture. There is a state variable viewport, which contains exactly this information. It is a pair, of a position and a size. The position is the offset from the upper left corner in pixels. The size is the size of the screen to be used for rendering in pixels.

Example:

If you want the window to be used completely for rendering the image, then the

position needs to be set to Position 0 0. i.e. no offset and as size the complete window size is to be used:

```
_ Reshape2.hs _
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
3
    import PointsForRendering
4
\mathbf{5}
    main = do
6
      (progName,_) <- getArgsAndInitialize</pre>
7
      createWindow progName
8
      displayCallback $= display
9
      reshapeCallback $= Just reshape
10
      mainLoop
11
12
    display = do
13
      clear [ColorBuffer]
14
      displayPoints points Quads
15
       where
16
        points
17
          = [(0.5, 0.5, 0)]
18
            , (-0.5, 0.5, 0)
19
            ,(-0.5,-0.5,0)
20
            ,(0.5,-0.5,0)]
^{21}
^{22}
    reshape s@(Size w h) = do
23
      viewport $= (Position 0 0, s)
^{24}
```

If you start this program and resize the window, then always the complete window pane will be used for rendering your image.

Example:

In this example only parts of the window are used for rendering the image. The image is smaller than the window.

```
– Viewport.hs —
    import Graphics.UI.GLUT
1
    import Graphics.Rendering.OpenGL
2
3
    import PointsForRendering
4
\mathbf{5}
    main = do
6
      (progName,_) <- getArgsAndInitialize</pre>
7
      createWindow progName
8
      clearColor $= Color4 0 0 0 0
9
      displayCallback $= display
10
      reshapeCallback $= Just reshape
11
      mainLoop
12
13
   display = do
14
```

15	clearColor \$= Color4 1 1 1 1
16	clear [ColorBuffer]
17	currentColor \$= Color4 1 0 0 1
18	displayPoints ps1 LineLoop
19	displayPoints ps2 Lines
20	where
21	ps1=[(0.5,0.5,0),(-0.5,0.5,0),(-0.5,-0.5,0),(0.5,-0.5,0)]
22	ps2=[(1,1,0),(-1,-1,0),(-1,1,0),(1,-1,0)]
23	
24	reshape s@(Size w h) = do
25	viewport \$= (Position 50 50, Size (w-80) (h-60))

The resulting window can be found in figure 4.2.

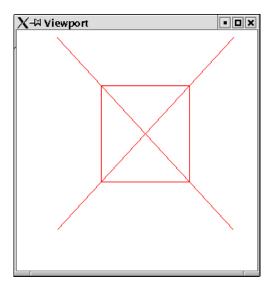


Figure 4.2: A Viewport smaller than the window.

4.3 Orthographic Projection

The viewport defines which parts of your window pane are used for rendering your image. The actual projection defines which coordinates you want to display. The simpliest way to specify this is by the function ortho. It has six arguments, the lower and upper bounds of the x, y, z coordinates.

Projection is equally as transformation internally expressed in terms of a matrix. The statement loadIdentity can refer to the transformation or to the projection matrix. A state variabble matrixMode defines, which of these matrixes these statements refer to. Therefore it is necessary to switch this variable to the value Projection, before applying the function ortho and afterwards to reset the variable back to the value ModelView.

Example:

We render the same image in two windows with different projection values:

```
- Ortho.hs
    import PointsForRendering
1
    import Graphics.Rendering.OpenGL
^{2}
3
    import Graphics.UI.GLUT as GLUT
    import Star
4
\mathbf{5}
    main = do
6
      (progName,_) <- getArgsAndInitialize</pre>
7
8
      createWindow (progName++"1")
9
      displayCallback $= display
10
      projection (-5) 5 (-5) 5 (-5) 5
11
12
13
      createWindow (progName++"2")
      displayCallback $= display
14
      projection 0 0.8 (-0.8) 0.8 (-0.5) 0.5
15
16
      mainLoop
17
18
    projection xl xu yl yu zl zu = do
19
      matrixMode $= Projection
20
      loadIdentity
^{21}
      ortho xl xu yl yu zl zu
^{22}
      matrixMode $= Modelview 0
23
^{24}
    display = do
^{25}
     clearColor $= Color4 1 1 1 1
26
     clear [ColorBuffer]
27
     currentColor $= Color4 1 0 0 1
^{28}
     star 0.9 7
29
     currentColor $= Color4 1 1 0 1
30
     star 0.4 5
31
```

The resulting windows can be found in figure 4.3.

ortho is the simpliest projection we can define. When we will consider third dimensional szeneries we will learn a more powerful projection.

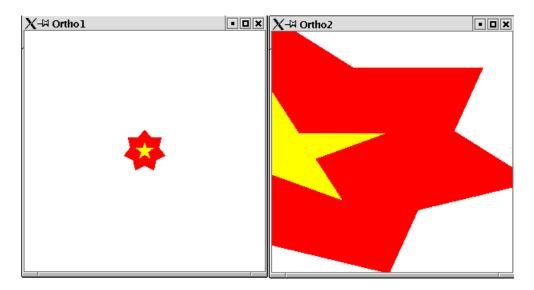


Figure 4.3: Two windows with different projection.

Chapter 5

Changing States

OpenGL is not only designed to render static images, but to have changing images. There are to ways how your image might change:

- it might react to some event, like some keyboard input or mouse event.
- it might change over time.

In order to change your image in some coordinated way, you need a state which can change. An event may change your state, or over the time your state might be changed.

5.1 Modelling your own State

A state is of course something, which does not match the purely functional paradigm of Haskell. However in the context of I/O the designers of Haskell came up with some clever way to integrate state changing variables into the Haskell's purely functional setting. The trick are again monads, as you have seen before for the state machine of OpenGL. There is a standard library in Haskell for state changing variables: Data.IORef. This provides functions for creation, setting, retrieving and modification of state variables. These functions are called:

```
newIORef, writeIORef, readIORef, modifyIORef.
```

If you think these names a bit too technical, then you might use the following module, which makes IORef variables instances of the type classes HasGetter and HasSetter. Thus we can use our own state variables in the same way, we use the HOpenGL state variables.¹

```
StateUtil.hs -
module StateUtil where
import Graphics.Rendering.OpenGL
import Data.IORef
import Graphics.UI.GLUT
```

¹This is no longer necessary, since this has been integrated into the HOpenGL library.

```
7 --instance HasSetter IORef where
8 -- ($=) var val = writeIORef var val
9
10 --instance HasGetter IORef where
11 -- get var = readIORef var
12
13 new = newIORef
```

5.2 Handling of Events

Now we know how to modell our own state. We can use this for reacting on some events. Event handling in HOpenGL is done by setting a callback function for mouse and keyboard events. A callback function for mouse and keyboard events needs to be of the following type:

```
type KeyboardMouseCallback =
   Key -> KeyState -> Modifiers -> Position -> IO ()
```

A Key can be some character, some special character or some mouse buttom:

```
1 data Key
2 = Char Char
3 | SpecialKey SpecialKey
4 | MouseButton MouseButton
5 deriving (Eq, Ord, Show)
```

1

2

1

2

The keystate informs, if the key has been pressed or released.

```
1 data KeyState
2 = Down
3 | Up
4 deriving ( Eq, Ord, Show )
```

A modifier denotes, if some extra key is used, like the alt, strg or shift key:

```
data Modifiers = Modifiers { shift, ctrl, alt :: KeyState }
    deriving ( Eq, Ord, Show )
```

And finally the position informs about the current mouse pointer position.

5.2.1 Keyboard events

With the close look at the event handling function above it is fairly easy to write a program that reacts on keyboard events. A function of type KeyboardMouseCallback is to be written and assigned to the state variable keyboardMouseCallback of your window. Usually your

KeyboardMouseCallback will have access to some of your state variables, since you want to change a state when an event occurs. When the state has been changed, HOpenGL needs to be forced to redisplay the picture with the new state values. Therefore a call to the function postRedisplay needs to be done.

Example:

In this example we draw a circle. The radius of the circle can be changed by use of the + and - key.

```
_ State.hs _
    import Circle
1
    import PointsForRendering
2
    import StateUtil
3
4
    import Graphics.Rendering.OpenGL
\mathbf{5}
    import Data.IORef
6
    import Graphics.UI.GLUT
7
8
    main = do
9
      (progName,_) <- getArgsAndInitialize</pre>
10
      createWindow progName
11
```

We create a state variable which stores the current radius of the circle:

_ State.hs .

_ State.hs _

radius <- new 0.1 12

The display function gets this state variable as first argument:

```
- State.hs
     displayCallback $= display radius
13
```

And the keyboard callback gets this variable as first argument:

14 15

16

17

keyboardMouseCallback \$= Just (keyboard radius) mainLoop

The display function gets the current value for the radius and draws a filled circle:

```
_ State.hs _
display radius = do
  clear [ColorBuffer]
```

```
r <- get radius
18
19
```

fillCircle r

The keyboard callback reacts on two keyboard events. The value of the radius variable are changed:

```
_ State.hs _
    keyboard radius (Char '+') Down _ _ = do
20
^{21}
      r <- get radius
      radius $= r+0.05
22
```

```
23 postRedisplay Nothing
24 keyboard radius (Char '-') Down _ _ = do
25 r <- get radius
26 radius $= r-0.05
27 postRedisplay Nothing
28 keyboard _ _ _ = return ()
```

Compile and start this program and press the + and - key.

5.3 Changing State over Time

The second way to change your picture is over time. You can create an animation if your picture changes a tiny bit every moment. In HOpenGL you can a define a so called *idle* function. This function will be evaluated whenever the picture has been displayed. There you can define, in what way your state will change before the next redisplay is performed. The last statement in an *idle* function will be usually a call to **postRedisplay**.

Example:

We define our first animation. A ring is displayed with a changing radius.

```
Idle.hs

import Ring

import PointsForRendering

import StateUtil

import Graphics.Rendering.OpenGL

import Data.IORef

import Graphics.UI.GLUT as GLUT
```

We define a constant which denotes the value by which the radius changes between every redisplay:

_ Idle.hs _

_ Idle.hs

STEP = 0.001

8

9

Within the main function an *idle* callback is added to the window:

```
main = do
10
      (progName,_) <- getArgsAndInitialize</pre>
11
      createWindow progName
12
      radius <- new 0.1
13
      step
              <- new _STEP
14
      displayCallback $= display radius
15
      idleCallback $= Just (idle radius step)
16
      mainLoop
17
```

The display function renders a ring, depending on the state variable for the radius:

```
18 display radius = do
19 clear [ColorBuffer]
20 r <- get radius
21 ring r (r+0.2)
22 flush
```

The idle function changes the value of the variable **radius** depending on the second state variable **step**.

_ Idle.hs _

```
_ Idle.hs .
    idle radius step = do
23
      r <- get radius
^{24}
      s <- get step
25
      if r>=1 then step $= (-_STEP)
26
               else if r<=0 then step $= _STEP
27
                              else return ()
28
      s <- get step
29
      radius $= r+s
30
      postRedisplay Nothing
31
```

5.3.1 Double buffering

The animation created in the last example was not very satisfactory. A ring with changing radius was displayed, but the animation was somehow flickering. The reason for that was, that the display function as its first statement clears the screen, i.e. makes it alltogether black. Only afterwards the ring is rendered. For a short moment the screen will be completely black. This is what makes this flickering effect.

A common solution for this problem in animated pictures is, not to apply the statements of the display function directly to the screen, but to an invisible buffer. When all statements of the display function have been applied to this invisible background buffer, this buffer is copied to the screen. This way only the ready to use final picture is shown on screen and not any intermediate rendering step (e.g. the picture after the clear statement).

OpenGL provides a double buffering mechanism. We only have to activate this. Therefore we need to set the initial display mode variable accordingly. Instead of a call to the function flush a call to the function swapBuffers needs to be done as last statement of the display function.

Example:

The ring with changing radius over time now with double buffering.

```
Double.hs

import Ring

import PointsForRendering

import StateUtil

import Graphics.Rendering.OpenGL

import Data.IORef

import Graphics.UI.GLUT as GLUT
```

```
8
    STEP = 0.001
9
10
    main = do
11
      (progName,_) <- getArgsAndInitialize</pre>
12
      initialDisplayMode $= [DoubleBuffered]
13
      createWindow progName
14
      radius <- new 0.1
15
              <- new _STEP
      step
16
      displayCallback $= display radius
17
      idleCallback $= Just (idle radius step)
18
      mainLoop
19
20
    display radius = do
^{21}
      clear [ColorBuffer]
^{22}
      r <- get radius
23
      ring r (r+0.2)
^{24}
25
      swapBuffers
26
    idle radius step = do
27
      r <- get radius
28
      s <- get step
29
      if r>=1 then step $= (-_STEP)
30
               else if r<=0 then step $= _STEP
31
                              else return ()
32
      s <- get step
33
      radius $= r+s
^{34}
      postRedisplay Nothing
35
```

5.4 Pong: A first Game

By now you have seen a lot of tiny examples. It is time to draw the techniques together and do an application with HOpenGL. In this section we will implement one of the first animated computer games ever: Pong. It consists of a small white circle which moves over a black screen and two paddles which can move on a vertical line.

Pong in action can be found in figure 5.1.

```
_ Pong.hs _
```

```
import Circle
import Squares
import PointsForRendering
import StateUtil
import Graphics.Rendering.OpenGL
import Data.IORef
import Graphics.UI.GLUT as GLUT
```

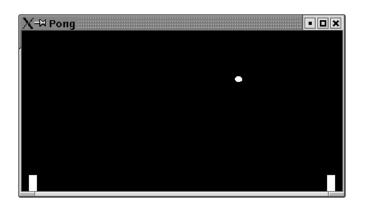


Figure 5.1: Pong in action.

First of all we define some constant values for the game: x-, y-coordinates of the game, width and height of a paddle, the radius of the ball, initial factor, how a ball and a paddle changes its position, and an initial board size.

___ Pong.hs __

```
LEFT
            = -2
9
                1
10
    _RIGHT =
    TOP
            =
               1
11
    BOTTOM= -1
12
13
    paddleWidth = 0.07
14
    paddleHeight = 0.2
15
    ballRadius
                   = 0.035
16
17
    _INITIAL_WIDTH :: GLsizei
^{18}
    _INITIAL_WIDTH=400
19
^{20}
    _INITIAL_HEIGHT::GLsizei
^{21}
    _INITIAL_HEIGHT=200
^{22}
23
    _INITIAL_BALL_DIR = 0.002
^{24}
    _INITIAL_PADDLE_DIR = 0.005
^{25}
```

We define a data type, game. The game state can be characterized by the position of the ball and the values these coordinates change for the next redisplay:

			Pong.hs	
26	data Ball	= Ball	(GLfloat,GLfloat) GLfloat GLfloat	

The paddles, which are characterized by their position and the position change on the y-axis (x-axis is fixed for a paddle).

	Pong.hs	
27	type Paddle = (GLfloat,GLfloat,GLfloat)	

Additionally a game has points for the left and the right player and a factor which denotes how fast ball and paddles move:

_ Pong.hs

```
28 data Game
29 = Game { ball ::Ball
30 , leftP,rightP :: Paddle
31 , points ::(Int,Int)
32 , moveFactor::GLfloat}
33
```

For a starting game we provide the following initial game state:

```
_____ Pong.hs ____
```

```
initGame
34 initGame
35 = Game {ball=Ball (-0.8,0.3) _INITIAL_BALL_DIR _INITIAL_BALL_DIR
36 ,leftP=(_LEFT+paddleWidth,_BOTTOM,0)
37 ,rightP=(_RIGHT-2*paddleWidth,_BOTTOM,0)
38 ,points=(0,0)
39 ,moveFactor=1
40 }
```

The main function creates a double buffering window in fullscreen mode. An initial game state is created and passed to the keyboard, display, idle and reshape function:

Pong.hs

```
main = do
41
      (progName,_) <- getArgsAndInitialize
42
      initialDisplayMode $= [DoubleBuffered]
^{43}
      createWindow progName
44
      game <- newIORef initGame</pre>
^{45}
      --windowSize $= Size _INITIAL_WIDTH _INITIAL_HEIGHT
^{46}
      fullScreen
47
      displayCallback $= display game
48
      idleCallback $= Just (idle game)
49
      keyboardMouseCallback $= Just (keyboard game)
50
      reshapeCallback $= Just (reshape game)
51
      mainLoop
52
```

The display function simply gets the ball and paddles from the game state and renders these:

_ Pong.hs -

```
display game = do
53
      clear [ColorBuffer]
54
      g <- get game
55
      let (Ball pos xDir yDir) = ball g
56
      --a ball is a circle
57
      displayAt pos $ fillCircle ballRadius
58
      displayPaddle$leftP g
59
      displayPaddle$rightP g
60
      swapBuffers
61
```

Paddles are simply rectangles:

```
Pong.hs ______

62 displayPaddle (x,y,_) = preservingMatrix$do

63 translate$Vector3 (paddleWidth/2) (paddleHeight/2) 0

64 displayAt (x,y)$myRect paddleWidth paddleHeight
```

We made use of the utility function which moves a shape to some position:

```
Pong.hs ______

65 displayAt (x, y) displayMe = preservingMatrix$do

66 translate$Vector3 x y (0::GLfloat)

67 displayMe
```

Within the idle function ball and paddles need to be set to their next position on the field:

```
_____ Pong.hs _
    idle game = do
68
      g <- get game
69
      let fac = moveFactor g
70
      game
71
        $= g{ball
                     = moveBall g
72
             ,leftP = movePaddle (leftP g) fac
73
             ,rightP = movePaddle (rightP g) fac
74
75
      postRedisplay Nothing
76
```

The movement on the ball is determined by the upper and lower bound of the field, by the left and right bound of the field and the position of the paddles:

```
_ Pong.hs _
    moveBall g
77
     = Ball (x+factor*newXDir,y+factor*newYDir) newXDir newYDir
78
      where
79
       newXDir
80
                x-ballRadius <= xl+paddleWidth
81
         && y+ballRadius >=yl
82
                               <=yl+paddleHeight
           && Y
83
            = -xDir
84
         |x <= _LEFT-ballRadius = 0</pre>
85
               x+ballRadius >= xr
86
           && y+ballRadius >=yr
87
           && У
                               <=yr+paddleHeight
88
           = -xDir
89
         x >= _RIGHT+ballRadius = 0
90
         otherwise
                     = xDir
91
       newYDir
92
         |y > _TOP-ballRadius || y< _BOTTOM+ballRadius = -yDir</pre>
93
         |\text{newXDir} == 0 = 0
94
         otherwise = yDir
95
```

```
96 (Ball (x,y) xDir yDir) = ball g
97 factor = moveFactor g
98 (xl,yl,_) = leftP g
99 (xr,yr,_) = rightP g
100
```

A paddle moves only on the y-axis. We just need to ensure that it does not leaves the field. There are maximum and minimum values for y:

```
Pong.hs

movePaddle (x,y,dir) factor =

let y1 = y+ factor*dir

newY = min (_TOP-paddleHeight) $max _BOTTOM y1

in (x,newY,dir)
```

The keyboard function: key 'a' moves the left paddle, key 'l' the right paddle and the space key gets a new ball:

____ Pong.hs _

```
105
    keyboard game (Char 'a') upDown _ _ = do
106
      g <- get game
107
      let (x,y,_) = leftP g
108
      game $= g{leftP=(x,y,paddleDir upDown)}
109
    keyboard game (Char 'l') upDown _ _ = do
110
      g <- get game
111
      let (x,y,_) = rightP g
112
      game $= g{rightP=(x,y,paddleDir upDown)}
113
    keyboard game (Char '\32') Down _ _ = do
114
      g <- get game
115
      let Ball (x,y) xD yD = ball g
116
      let xDir
117
             |x<=_LEFT+3*paddleWidth = _INITIAL_BALL_DIR</pre>
118
            x>=_RIGHT-3*paddleWidth = - _INITIAL_BALL_DIR
119
            |otherwise = xD
120
       if (xD==0)
121
         then game$=g{ball=Ball (x+4*xDir,y) xDir INITIAL BALL DIR}
122
         else return ()
123
    keyboard \_ \_ \_ \_ = return ()
124
125
    paddleDir Down = _INITIAL_PADDLE_DIR
126
    paddleDir Up = -_INITIAL_PADDLE_DIR
127
```

Finally we define the visual part of the screen. The movement factor of the ball depends on the width of the screen:

Pong.hs reshape game s@(Size w h) = do viewport \$= (Position 0 0, s) matrixMode \$= Projection

```
131 loadIdentity
132 ortho (-2.0) 1.0 (-1.0) 1.0 (-1.0) 1.0
133 matrixMode $= Modelview 0
134 g <- get game
135 game$=g{moveFactor=fromIntegral w/fromIntegral _INITIAL_WIDTH}</pre>
```

Have a break and play *Pong*.

Chapter 6

Third Dimension

Up to now everything was pretty boring. We never considered the three dimensional space provided by OpenGL. Strictly we just considered two dimensions. Thus the library was not any more powerfull than any simple graphics libaray e.g. like Java's java.awt.Graphics class. In this chapter we will explore the true power of OpenGL by actually rendering three dimensional objects.

6.1 Hidden Shapes

In a three dimensional space some objects will be in front of others and hide them. We would expect to see only those areas which are not hidden by areas closer to the viewer.

Example:

We render two shapes. A red square which is closer to the viewer and a blue circle which is farer away:

```
_ NotHidden.hs
    import Graphics.Rendering.OpenGL
1
    import Graphics.UI.GLUT as GLUT
2
    import Squares
3
    import Circle
4
    import PointsForRendering
5
6
    main = do
7
      (progName,_) <- getArgsAndInitialize</pre>
8
      createWindow progName
9
      displayCallback $= display
10
      clearColor $= Color4 1 1 1 1
11
      mainLoop
12
13
    display = do
14
      clear [ColorBuffer,DepthBuffer]
15
      loadIdentity
16
      translate (Vector3 0 0 (-0.5::GLfloat))
17
```

```
18 currentColor $= Color4 1 0 0 1
19 square 1
20
21 loadIdentity
22 translate (Vector3 0.2 0.2 (0.5::GLfloat))
23 currentColor $= Color4 0 0 1 1
24 fillCircle 0.5
25 flush
```

However as can be seen in figure 6.1, the blue circle hides parts of the red square.

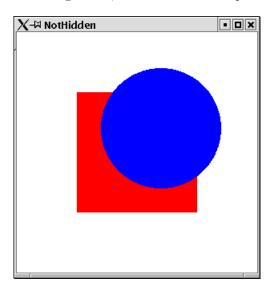


Figure 6.1: Third dimension not correctly taken into account.

By default OpenGL does not take the depth into account. Shapes rendered later hide other shapes which were rendered earlier, neglecting the depth of the shapes. OpenGL provides a mechanism for automatically considering the depth of a shape. This simply needs to be activated. Three steps need to be done:

- as initial display mode WithDepthBuffer needs to be set.
- a depth function needs to be set. Usually the Less mode is used as function here. This ensures that closer objects hide objects farer away.
- the depth buffer needs to be cleared in the beginning of the display function.

Example:

Now we render the same to shapes as in the example before, but the depth machanism of OpenGL is activated.

```
Hidden.hs

import Graphics.Rendering.OpenGL

import Graphics.UI.GLUT as GLUT

3
```

```
import Squares
4
    import Circle
\mathbf{5}
6
    import PointsForRendering
7
    main = do
8
      (progName,_) <- getArgsAndInitialize</pre>
9
      initialDisplayMode $= [WithDepthBuffer]
10
11
      createWindow progName
12
^{13}
      depthFunc $= Just Less
14
      displayCallback $= display
15
      clearColor $= Color4 1 1 1 1
16
      mainLoop
17
18
    display = do
^{19}
      clear [ColorBuffer,DepthBuffer]
^{20}
      loadIdentity
^{21}
      translate (Vector3 0 0 (-0.5::GLfloat))
22
      currentColor $= Color4 1 0 0 1
23
      square 1
^{24}
25
      loadIdentity
26
      translate (Vector3 0.2 0.2 (0.5::GLfloat))
27
      currentColor $= Color4 0 0 1 1
28
      fillCircle 0.5
29
      flush
30
```

Now as can be seen in figure 6.2, the red square hides parts of the blue circle.

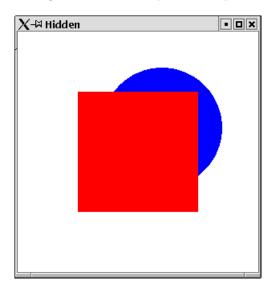


Figure 6.2: Third dimension correctly taken into account by use of depth function.

6.2 Perspective Projection

In the real world objects closer to the viewer appear larger than objects farer away from the viewer. Up to now we only learnt how to set up an orthographic projection. In an orthographic projection objects farer away have the same size as object close to the viewer.

Example:

```
import Graphics.Rendering.OpenGL
1
    import Graphics.UI.GLUT as GLUT
^{2}
з
    import Squares
4
    import Circle
5
    import PointsForRendering
6
7
    main = do
8
      (progName, ) <- getArgsAndInitialize
9
      initialDisplayMode $= [WithDepthBuffer]
10
      createWindow progName
11
      depthFunc $= Just Less
12
      displayCallback $= display
13
14
      matrixMode $= Projection
15
      loadIdentity
16
      ortho (-5) 5 (-5) 5 (1) 40
17
      matrixMode $= Modelview 0
18
19
      clearColor $= Color4 1 1 1 1
20
      mainLoop
^{21}
22
    display = do
23
      clear [ColorBuffer,DepthBuffer]
^{24}
      loadIdentity
25
      translate (Vector3 0 0 (-2::GLfloat))
26
      currentColor $= Color4 1 0 0 1
27
      square 1
28
29
      loadIdentity
30
      translate (Vector3 4 4 (-5::GLfloat))
31
      currentColor $= Color4 0 0 1 1
32
      square 1
33
      flush
34
```

As can be seen in figure 6.3 , the two squares have the same size, even though the red one is closer to the viewer.

OpenGL provides the function frustum for specifying a perspective projection. frustum has 6 arguments:

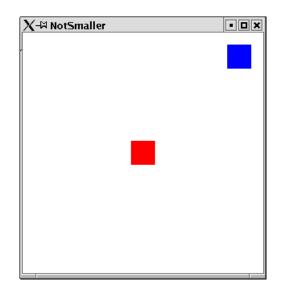


Figure 6.3: Two squares in orthographic projection.

- *left*: left bound for the closest orthogonal plane
- *right*: right bound for the closest orthogonal plane
- *top*: upper bound for the closest orthogonal plane
- *bottom*: lower bound for the closest orthogonal plane
- *near*: the closest things that can be seen
- far: the farest away things that can be seen

Figure 6.4 illustrates these six values.

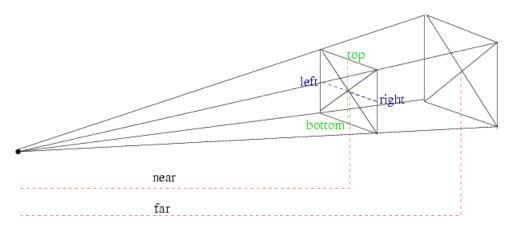


Figure 6.4: Perspective projection with frustum.

Usually you will have negated values for top/botton and left/right.

Example:

Now we render the two squares from the previous example again. This time we use a perspective projection:

```
_ Smaller.hs
    import Graphics.Rendering.OpenGL
1
    import Graphics.UI.GLUT as GLUT
2
3
    import Squares
4
    import Circle
\mathbf{5}
    import PointsForRendering
6
7
    main = do
8
      (progName,_) <- getArgsAndInitialize</pre>
9
      initialDisplayMode $= [WithDepthBuffer]
10
      createWindow progName
11
      depthFunc $= Just Less
12
      displayCallback $= display
13
14
      matrixMode $= Projection
15
      loadIdentity
16
      let near = 1
17
                 = 40
           far
18
           right = 1
19
                  = 1
           top
20
      frustum (-right) right (-top) top near far
^{21}
      matrixMode $= Modelview 0
22
23
      clearColor $= Color4 1 1 1 1
^{24}
      mainLoop
^{25}
^{26}
    display = do
27
      clear [ColorBuffer,DepthBuffer]
28
      loadIdentity
29
      translate (Vector3 0 0 (-2::GLfloat))
30
      currentColor $= Color4 1 0 0 1
31
      square 1
32
33
      loadIdentity
34
      translate (Vector3 4 4 (-5::GLfloat))
35
      currentColor $= Color4 0 0 1 1
36
      square 1
37
      flush
38
```

Now, as can be seen in figure 6.5, the blue square appears to be smaller than the red square.

HopenGL provides a second function to define a perspective projection: **perspective**. Here instead of left, right, top, bottom an angle between the top/bottom ray and the width of the closest plane can be specified.

Figure 6.6 illustrates these values.

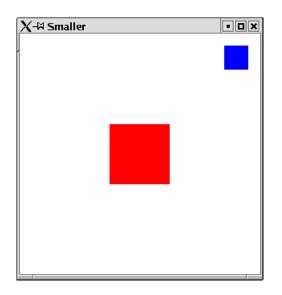


Figure 6.5: Two squares in perspective projection.

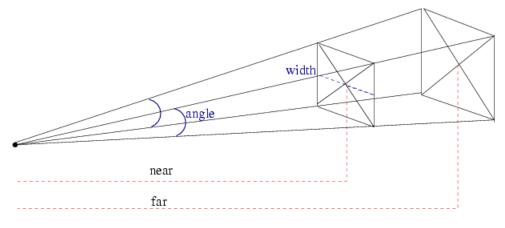


Figure 6.6: Perspective projection.

6.3 Setting up the Point of View

In the previous section we have learnt that there is a second way how to project the three dimensional space onto the two dimensional area of the screen. We did however not yet specify, where in the three dimensional space the viewer is situated and in what direction they are looking. In order to define this, OpenGL provides the function lookAt. It has three arguments:

- the point, where the viewer is situated.
- the point at which the viewer is looking.
- and a vector, which specifies the direction which is to be up for the viewer.

6.3.1 Oribiting around the origin

The point of view, where we are looking from is interesting, when we change it. In the following a module is defined, which allows the viewer to move along a sphere. The point of view can be set for a given sphere position. The position is specified by two angles and a radius. The first angle defines which way to move around the x-axis the second angle, which angle to move around the y-axis. The radius defines the distance from the origin. The position can be changed through keyboard events.

```
OrbitPointOfView.hs -
   module OrbitPointOfView where
1
\mathbf{2}
    import Graphics.Rendering.OpenGL
3
    import Graphics.UI.GLUT as GLUT
4
5
    import StateUtil
6
    import Data.IORef
7
8
    setPointOfView pPos = do
9
      (alpha,beta,r) <- get pPos
10
      let
11
                   = calculatePointOfView alpha beta r
       (x,y,z)
12
       (x2,y2,z2) = calculatePointOfView ((alpha+90)'mod' 360) beta r
13
      lookAt (Vertex3 x y z) (Vertex3 0 0 0) (Vector3 x2 y2 z2)
14
15
    calculatePointOfView alp bet r =
16
      let alpha = fromIntegral alp*2*pi/fromIntegral 360
17
          beta = fromIntegral bet*2*pi/fromIntegral 360
18
          y = r * \cos alpha
19
          u = r * sin alpha
20
          x = u * cos beta
21
          z = u * sin beta
22
      in (x,y,z)
23
^{24}
   keyForPos pPos (Char '+')
                                         = modPos pPos (id, id, x \rightarrow x - 0.1)
^{25}
   keyForPos pPos (Char '-')
                                         = modPos pPos (id, id, (+)0.1)
26
   keyForPos pPos (SpecialKey KeyLeft) = modPos pPos (id,(+)359,id)
27
   keyForPos pPos (SpecialKey KeyRight) = modPos pPos (id,(+)1,id)
28
   keyForPos pPos (SpecialKey KeyUp)
                                         = modPos pPos ((+)1,id,id)
^{29}
   keyForPos pPos (SpecialKey KeyDown) = modPos pPos ((+)359,id,id)
30
   keyForPos
                                           = return ()
31
32
   modPos pPos (ffst,fsnd,ftrd) = do
33
      (alpha,beta,r) <- get pPos</pre>
34
      pPos $= (ffst alpha 'mod' 360,fsnd beta 'mod' 360,ftrd r)
35
      postRedisplay Nothing
36
37
   reshape screenSize@(Size w h) = do
38
      viewport $= ((Position 0 0), screenSize)
39
      matrixMode $= Projection
40
      loadIdentity
41
```

```
let near
                  = 0.001
42
                  = 40
43
          far
          fov
                  = 90
44
          ang
                  = (fov*pi)/(360)
45
                  = near / ( cos(ang) / sin(ang) )
          top
46
          aspect = fromIntegral(w)/fromIntegral(h)
47
          right = top*aspect
48
      frustum (-right) right (-top) top near far
49
      matrixMode $= Modelview 0
50
```

Example:

Let us use the module above, to orbit around a cube. Therefore we a define simple module, which renders a cube with differently colored areas. The cube is situated at the origin. We render the six areas by rendering a square at the origin and translate and rotate it into its final position.

```
_ ColorCube.hs
    module ColorCube where
1
2
    import Graphics.Rendering.OpenGL
3
    import Graphics.UI.GLUT as GLUT
4
\mathbf{5}
    import Squares
6
    import StateUtil
7
8
    locally = preservingMatrix
9
10
    colorCube n = do
11
      locally $ do
12
        currentColor $= Color4 1 0 0 1
13
        translateVector3 \ 0 \ (-n/2)
14
        square n
15
      locally $ do
16
        currentColor $= Color4 0 1 0 1
17
        translate$Vector3 0 0 (n/2)
18
        square n
19
      locally $ do
20
        currentColor $= Color4 0 0 1 1
21
        translate$Vector3 (n/2) 0 0
22
        rotate 90 $Vector3 0 (1::GLfloat) 0
23
        square n
^{24}
      locally $ do
25
        currentColor $= Color4 1 1 0 1
26
        translate\Vector3 (-n/2) 0 0
27
        rotate 90 $Vector3 0 (1::GLfloat) 0
28
        square n
^{29}
      locally $ do
30
        currentColor $= Color4 0 1 1 1
31
        translate\Vector3 0 (-n/2) 0
32
        rotate 90 $Vector3 (1::GLfloat) 0 0
33
```

```
    34
    square n

    35
    locally $ do

    36
    currentColor $= Color4 1 1 1 1

    37
    translate$Vector3 0 (n/2) 0

    38
    rotate 90 $Vector3 (1::GLfloat) 0 0

    39
    square n
```

The following program allows to use the cursor keys to move around a cube at the origin:

```
OrbitAroundCube.hs
    import Graphics.Rendering.OpenGL
1
    import Graphics.UI.GLUT as GLUT
2
3
    import Squares
4
    import OrbitPointOfView
5
    import StateUtil
6
    import ColorCube
7
8
9
    main = do
      (progName,_) <- getArgsAndInitialize</pre>
10
      initialDisplayMode $= [WithDepthBuffer,DoubleBuffered]
11
      createWindow progName
12
      depthFunc $= Just Less
13
14
      pPos <- new (90::Int,270::Int,2.0)
15
      keyboardMouseCallback $= Just (keyboard pPos)
16
17
      displayCallback $= display pPos
18
      reshapeCallback $= Just reshape
19
      mainLoop
20
```

The display function sets the viewer's position before rendering the cube:

```
OrbitAroundCube.hs
display pPos = do
loadIdentity
setPointOfView pPos
clear [ColorBuffer,DepthBuffer]
colorCube 1
swapBuffers
```

As keyboard function we map directly to the function defined in OrbitPointOfView.

```
OrbitAroundCube.hs _____
27 keyboard pPos c _ _ = keyForPos pPos c
```

An example how the colored cube can now be seen is given in figure 6.7.

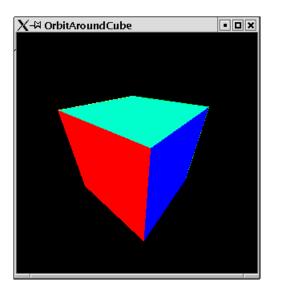


Figure 6.7: A view of the colored cube,

6.4 3D Game: Rubik's Cube

In this section we implement a primitive version of Rubik's cube. Rubik's cube in action can be found in figure 6.8.

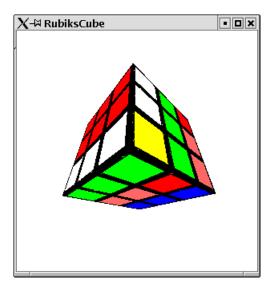


Figure 6.8: Rubik's Cube in action.

6.4.1 Cube Logics

First of all we modell the logics of Rubic's Cube¹.

A data type is provided for representation of a cube:

```
_ RubikLogic.hs _
   module RubikLogic where
1
2
   data Rubik a
3
     = Rubik (Front a) (Top a) (Back a) (Bottom a) (Left a) (Right a)
4
\mathbf{5}
   type Front a = Area a
6
    type Top a
                   = Area a
7
    type Back a
                   = Area a
8
    type Bottom a = Area a
9
10
    type Left a
                   = Area a
11
    type Right a = Area a
12
   type Area a = [Row a]
13
   type Row a = [a]
14
15
   data AreaPosition = Front |Top| Back| Bottom| Left| Right
16
17
   data RubikColor = Red|Blue|Yellow|Green|Orange|White|Black
18
```

We make the type Rubik an instance of the class Functor:

```
RubikLogic.hs

19 instance Functor Rubik where

20 fmap f (Rubik front top back bottom left right)

21 = Rubik (mf front) (mf top) (mf back)

22 (mf bottom) (mf left) (mf right)

23 where

24 mf = map (map f)
```

The initial cube is defined

```
RubikLogic.hs

25 initCube = Rubik (area Red) (area Blue) (area Yellow)

26 (area Green)(area Orange)(area White)

27

28 area c = [[c,c,c],[c,c,c]]
```

The main operation on a cube is to turn one of its six sides. The function rotateArea specifies, how this effects a cube.

¹There are certainly cleverer ways to do this, but I did not take the time to think of them.

```
_ RubikLogic.hs -
   rotateArea RubikLogic.Front
29
30
     (Rubik front top back bottom left right) =
     Rubik front' top' back bottom' left' right'
31
      where
32
         top'
                 = newRow 3 (reverse$column 3 left) top
33
         bottom' = newRow 1 (reverse$column 1 right) bottom
34
                = newColumn 3 (row 1 bottom) left
         left'
35
         right' = newColumn 1 (row 3 top) right
36
         front' = rotateBy3 front
37
38
   rotateArea RubikLogic.Back
39
     (Rubik front top back bottom left right) =
40
       Rubik front' top' back' bottom'
41
             (rotateBy2 left') (rotateBy2 right')
42
       where
^{43}
        (Rubik back' bottom' front' top' left' right') =
^{44}
          rotateArea RubikLogic.Front
^{45}
           (Rubik back bottom front top
46
                   (rotateBy2 left) (rotateBy2 right))
47
48
   rotateArea RubikLogic.Bottom
49
     (Rubik front top back bottom left right) =
50
     Rubik front' top back' bottom' left' right'
51
       where
52
         back′
                 = newRow 1 (reverse$row 3 left) back
53
         front' = newRow 3 (row 3 right) front
54
         left'
                 = newRow 3 (row 3 front) left
55
         right'
                 = newRow 3 (reverse$row 1 back) right
56
         bottom' = rotateBy1 bottom
57
58
59
   rotateArea RubikLogic.Top
60
     (Rubik front top back bottom left right) =
61
     Rubik front' top' back' bottom left' right'
62
       where
63
         back'
                 = newRow 3 (reverse$row 1 right) back
64
         front' = newRow 1 (row 1 left) front
65
                = newRow 1 (reverse$row 3 back) left
         left'
66
         right' = newRow 1 (row 1 front) right
67
         top' = rotateBy1 top
68
69
   rotateArea RubikLogic.Left
70
    (Rubik front top back bottom left right) =
71
     Rubik front' top' back' bottom' left' right
72
       where
73
         top'
                 = newColumn 1 (column 1 front) top
74
         bottom' = newColumn 1 (column 1 back) bottom
75
                 = rotateBy3 left
         left'
76
         back′
                 = newColumn 1 (column 1 top) back
77
```

```
= newColumn 1 (column 1 bottom) front
         front'
78
79
   rotateArea RubikLogic.Right
80
     (Rubik front top back bottom left right) =
81
      Rubik front' top' back' bottom' left right'
82
       where
83
                  = newColumn 3 (column 3 back) top
         top'
84
         bottom' = newColumn 3 (column 3 front) bottom
85
         right' = rotateBy3 right
86
                  = newColumn 3 (column 3 bottom) back
         back′
87
         front' = newColumn 3 (column 3 top) front
88
89
   rotateBy1
90
      [[x1,x2,x3]
91
      ,[x8,x,x4]
92
      , [x7, x6, x5]] =
93
      [[x3,x4,x5]
^{94}
      ,[x2,x,x6]
95
      ,[x1,x8,x7]]
96
97
   rotateBy2 = rotateBy1 .rotateBy1
98
   rotateBy3 = rotateBy2 .rotateBy1
99
```

Finally some useful functions for manipulation of an area are given.

```
_ RubikLogic.hs
    column n = map (\langle row - > row !! (n-1) \rangle
100
101
    row n area = area !!(n-1)
102
103
    newRow 1 row [a,r,ea] = [row,r,ea]
104
    newRow 2 row [a,r,ea] = [a,row,ea]
105
    newRow 3 row [a,r,ea] = [a,r,row]
106
107
    newColumn n column area = map (doIt n) areaC
108
       where
109
         areaC = zip area column
110
         doIt 1 ((r:ow),c) = c:ow
111
         doIt 2 ((r:o:w),c) = r:c:w
112
         doIt 3 ((r:o:w:xs),c) = r:o:c:xs
113
```

6.4.2 Rendering the Cube

We have a logical modell of a cube. Now we can render this in a coordinate system. In an earlier section we allready provided a function to render one single side. We simply need to render the six sides and move them to the correct position.

```
RenderRubik.hs
```

```
1 module RenderRubik where
2
```

```
import Graphics.Rendering.OpenGL as OpenGL hiding (Red, Green, Blue)
3
   import Graphics.UI.GLUT as GLUT hiding (Red,Green,Blue)
4
5
   import PointsForRendering
6
   import RubikLogic
7
   import RubikFace
8
    import Squares
9
10
    _FIELD_WIDTH :: GLfloat
11
    _FIELD_WIDTH = 1/3
12
13
   renderCube (Rubik front top back bottom left right) = do
14
    render RubikLogic.Top
                                top
15
    render RubikLogic.Back
                                back
16
    render RubikLogic.Front
                                front
17
    render RubikLogic.Bottom bottom
^{18}
    render RubikLogic.Left
                                left
19
    render RubikLogic.Right
20
                                right
21
   render Top
                  cs = preservingMatrix$do
22
      translate $Vector3 (1.5*_FIELD_WIDTH) 0 0
^{23}
      rotate (90)$Vector3 0 1 (0::GLfloat)
^{24}
      renderCubeSide cs
25
26
   render RubikLogic.Back  cs = preservingMatrix$ do
27
      translate $Vector3 0 0 (-1.5*_FIELD_WIDTH)
^{28}
      rotate (180)$Vector3 0 0 (1::GLfloat)
^{29}
      rotate (180)$Vector3 1 0 (0::GLfloat)
30
      renderCubeSide cs
31
32
   render Bottom cs = preservingMatrix$ do
33
      translate $Vector3 (-1.5*_FIELD_WIDTH) 0 0
^{34}
      rotate (270)$Vector3 0 1 (0::GLfloat)
35
      renderCubeSide cs
36
37
   render RubikLogic.Front cs = preservingMatrix$ do
38
      translate $Vector3 0 0 (1.5*_FIELD_WIDTH)
39
      renderCubeSide cs
^{40}
41
   render RubikLogic.Left cs = preservingMatrix$ do
^{42}
      translate $Vector3 0 (1.5*_FIELD_WIDTH) 0
43
      rotate (270) $Vector3 1 0 (0::GLfloat)
44
      renderCubeSide cs
45
^{46}
   render RubikLogic.Right cs = preservingMatrix$ do
47
      translate $Vector3 0 (-1.5*_FIELD_WIDTH)
^{48}
      rotate (270) $Vector3 1 0 (0::GLfloat)
49
      rotate (180)$Vector3 1 0 (0::GLfloat)
50
      renderCubeSide cs
51
52
```

```
53 renderCubeSide css = renderArea _FIELD_WIDTH css
54
55 field = square _FIELD_WIDTH
```

The following function maps our abstract color type to concrete OpenGL colors:

```
_ RenderRubik.hs _
   doColor Red
                  = Color4 1 0 0 1.0
56
   doColor Green = Color4 0 1 0 1.0
57
   doColor Blue = Color4 0 0 1 1.0
58
   doColor Yellow = Color4 1 1 0 1.0
59
   doColor Orange = Color4 1 0.5 0.5 1
60
   doColor White = Color4 1 1 1 1.0
61
   doColor Black = Color4 0 0 0 1.0
62
```

6.4.3 Rubik's Cube

Finally we can create a simple application.

```
_ RubiksCube.hs _
    import Graphics.Rendering.OpenGL
1
    import Graphics.UI.GLUT as GLUT
2
3
    import Data.IORef
4
\mathbf{5}
    import OrbitPointOfView
6
    import StateUtil
7
    import RubikLogic
8
    import RenderRubik
9
10
    main = do
11
      initialDisplayMode $= [DoubleBuffered,RGBMode,WithDepthBuffer]
12
      (progName,_) <- getArgsAndInitialize</pre>
13
14
      createWindow progName
15
16
      depthFunc $= Just Less
17
^{18}
      pPos <- new (90::Int,270::Int,2.0)
19
      pCube <- new initCube
20
^{21}
      displayCallback $= display pPos pCube
^{22}
      keyboardMouseCallback $= Just (keyboard pPos pCube)
^{23}
^{24}
^{25}
      reshapeCallback $= Just reshape
      mainLoop
26
27
    display pPos pCube = do
^{28}
      clearColor $= Color4 1 1 1 1
29
```

```
clear [ColorBuffer,DepthBuffer]
30
31
      loadIdentity
      setPointOfView pPos
32
      cube <- get pCube
33
      renderCube$fmap doColor cube
^{34}
      swapBuffers
35
36
   keyboard _ pCube (Char '1') Down _ _
37
      = rot pCube RubikLogic.Top
38
   keyboard _ pCube (Char '2') Down _ _
39
     = rot pCube RubikLogic.Bottom
40
   keyboard _ pCube (Char '3') Down _ _
41
^{42}
    = rot pCube RubikLogic.Front
   keyboard _ pCube (Char '4') Down _ _
^{43}
      = rot pCube RubikLogic.Back
44
   keyboard _ pCube (Char '5') Down _ _
^{45}
      = rot pCube RubikLogic.Left
^{46}
   keyboard _ pCube (Char '6') Down _ _
47
     = rot pCube RubikLogic.Right
48
   keyboard pPos _
                       С
49
      = keyForPos pPos c
50
51
   rot pCube p = do
52
      cube <- get pCube
53
      pCube $= rotateArea p cube
54
      postRedisplay Nothing
55
```

6.5 Light

Let us begin with a simple 3-dimensional shape: a cube. A cube has six squares which we can render as the primitive shape Quad.

```
_ Cube.hs _
    module Cube where
1
2
    import Graphics.Rendering.OpenGL
3
    import Graphics.UI.GLUT as GLUT
4
\mathbf{5}
    import PointsForRendering
6
7
    cube l = renderAs Quads corners
8
      where
9
       corners =
10
        [(1,0,1),(0,0,1),(0,1,1),(1,1,1)
11
        (1,1,1),(1,1,0),(1,0,0),(1,0,1)
12
        (0,0,0),(1,0,0),(1,0,1),(0,0,1)
13
        (1,1,0),(0,1,0),(0,0,0),(1,0,0)
14
        (0,1,1),(1,1,1),(1,1,0),(0,1,0)
15
```

16 17 , (0, 1, 1), (0, 1, 0), (0, 0, 0), (0, 0, 1)

Example:

]

We make a first try at rendering a cube:

	RenderCube.hs
1	import Graphics.Rendering.OpenGL
2	import Graphics.UI.GLUT as GLUT
3	
4	import Cube
5	
6	main = do
7	(progName,_) <- getArgsAndInitialize
8	createWindow progName
9	displayCallback \$= display
10	mainLoop
11	
12	display = do
13	clear [ColorBuffer]
14	rotate 40 (Vector3 1 1 (1::GLfloat))
15	cube 0.5
16	loadIdentity
17	flush

The resulting window can be found in figure 6.9. It is not very exiting, we see a white shape, which has the outline of a cube, but do not get the three dimensional visual effect of a cube.

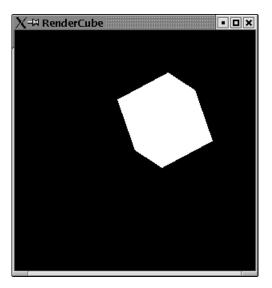


Figure 6.9: An unlit cube.

6.5.1 Defining a light source

For rendering three dimensional objects it is not enough to specify their shapes and your viewing position. Crucial is the way the objects are illuminated. In order to get a three dimensional viesual effect on your two dimensional computer screen, it needs to be defined what kind of light source lights the object.

A light source can be specified fairly easy. First you need to set the state variable lighting to the value Enabled. Then you need to specify the position of your light source. This can be done by setting a special position state variable, e.g. by

position (Light 0) \$= Vertex4 0.8 0 3.0 5.0.

And finally you need to turn the light source on by setting its state variable to enabled: light (Light 0) \$= Enabled.

Example:

Now we can render a cube with a defined light source:

```
_ LightCube.hs
    import Graphics.Rendering.OpenGL
1
    import Graphics.UI.GLUT as GLUT
\mathbf{2}
3
    import Cube
4
5
    main = do
6
      (progName, ) <- getArgsAndInitialize
7
8
      depthFunc $= Just Less
9
10
      createWindow progName
11
12
      lighting $= Enabled
13
      position (Light 0) $= Vertex4 1 0.4 0.8
                                                     1
14
      light (Light 0) $= Enabled
15
16
      displayCallback $= display
17
      mainLoop
18
19
    display = do
20
      clear [ColorBuffer]
^{21}
      rotate 40 (Vector3 1 1 (1::GLfloat))
22
      cube 0.5
23
      loadIdentity
^{24}
      flush
^{25}
```

The resulting window can be found in figure 6.10. Now we can identify a bit more the cube.

You might wonder, why the vertex for the light source position has four parameters. The forth parameter is a value by which the other three (the x, y, z coordinates) get divided.

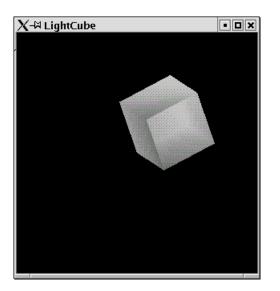


Figure 6.10: A lit cube.

6.5.2 Tux the Penguin

Let us render some cute object: Tux the penguin. We will roughly use the data from the OpenGL game tuxracer. The nice thing about a penguin is, that you can built it almost completely out of spheres. We will render Tux simply by rendering spheres, which are scaled to different forms and moved to the correct position.

Overall Setup

```
Tux.hs -
    import Graphics.Rendering.OpenGL as OpenGL
1
    import Graphics.UI.GLUT as GLUT
2
з
    import OrbitPointOfView
4
    import StateUtil
\mathbf{5}
6
   main = do
7
      (progName,_) <- getArgsAndInitialize</pre>
8
      initialDisplayMode $= [WithDepthBuffer,DoubleBuffered]
9
      pPos <- new (90::Int,270::Int,1.0)
10
      depthFunc $= Just Less
11
      createWindow progName
12
13
      lighting $= Enabled
14
      normalize $= Enabled
15
      depthFunc $= Just Less
16
17
      position (Light 0) $= Vertex4 0 0 (10) 0
18
19
      ambient (Light 0) $= Color4 1 1 1 1
      diffuse (Light 0) $= Color4 1 1 1 1
20
```

```
specular (Light 0) $= Color4 1 1 1 1
21
      light (Light 0) $= Enabled
^{22}
23
      displayCallback $= display pPos
^{24}
      keyboardMouseCallback $= Just (keyboard pPos)
25
      reshapeCallback $= Just reshape
^{26}
      mainLoop
27
28
   keyboard pPos c _ _ = keyForPos pPos c
29
```

The main display function clears the necessary buffers and calls the main function for rendering the penguin Tux.

_____ Tux.hs __

```
30 display pPos = do
31 loadIdentity
32 clearColor $= Color4 1 0 0 1
33 setPointOfView pPos
34 clear [ColorBuffer,DepthBuffer]
35 tux
36 swapBuffers
```

Auxilliary Functions

We will use some auxilliary functions. First of all a function, which renders a scaled sphere.

```
Tux.hs

sphere r xs ys zs = do

scal xs ys zs

createSphere r

createSphere r = renderObject Solid $Sphere' r 50 50

scal:: GLfloat -> GLfloat -> GLfloat -> IO ()

scal x y z = scale x y z
```

Furthermore some functions for easy translate and rotate transformations:

```
Tux.hs
    transl:: GLfloat -> GLfloat -> GLfloat -> IO ()
45
    transl x y z= translate$Vector3 x y z
46
47
   rota:: GLfloat -> GLfloat -> GLfloat -> GLfloat -> IO ()
^{48}
   rota a x y z = rotate a $ Vector3 x y z
^{49}
50
   rotateZ a = rota a 0 0 1
51
   rotateY a = rota a 0 1 0
52
   rotateX a = rota a 1 0 0
53
```

And eventually some functions to set the material properties for the different parts of a penguin.

```
Tux.hs
   crMat (rd,gd,bd) (rs,gs,bs) exp = do
54
      materialDiffuse
                        Front $= Color4 rd gd bd
                                                    1.0
55
     materialAmbient
                        Front $= Color4 rd gd bd
                                                    1.0
56
     materialSpecular Front $= Color4 rs gs bs
                                                   1.0
57
     materialShininess Front $= exp
58
59
     materialDiffuse
                        Back $= Color4 rd gd bd
                                                   1.0
60
     materialSpecular Back $= Color4 rs gs bs
                                                   1.0
61
     materialShininess Back $= exp
62
63
   whitePenguin = crMat (0.58, 0.58, 0.58)(0.2, 0.2, 0.2) 50.0
64
   blackPenguin = crMat (0.1, 0.1, 0.1)
                                          (0.5, 0.5, 0.5) 20.0
65
                = crMat (0.64, 0.54, 0.06)(0.4, 0.4, 0.4) 5
   beakColour
66
   nostrilColour= crMat (0.48039, 0.318627, 0.033725)(0.0,0.0,0.0) 1
67
   irisColour
                 = crMat (0.01, 0.01, 0.01)(0.4, 0.4, 0.4) 90.0
68
```

Torso and Head

The neck and torso of a penguin are almost black spheres with some white front parts. We will modell such figures by setting s white sphere in front of a black sphere.

_ Tux.hs _

```
makeBody = do
69
      preservingMatrix$do
70
        blackPenguin
71
        sphere 1 0.95 1.0 0.8
72
      preservingMatrix$do
73
        whitePenguin
74
        transl 0 0 0.17
75
        sphere 1 0.8 0.9 0.7
76
```

The resulting image can be found in figure 6.11.

Torso and shoulders are scaled body parts:

```
Tux.hs
    createTorso = preservingMatrix$do
77
      scal 0.9 0.9 0.9
78
      makeBody
79
80
    createShoulders = preservingMatrix$do
81
      transl 0 0.4 0.05
82
      leftArm
83
      rightArm
84
      scal 0.72 0.72 0.72
85
      makeBody
86
```

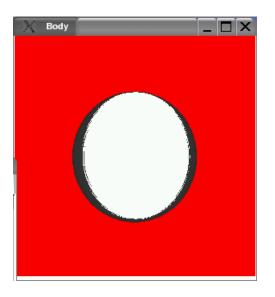


Figure 6.11: Basic part for a penguin torso.

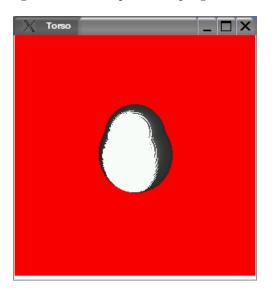


Figure 6.12: Penguin torso and shoulders.

The resulting image for torso and shoulders can be found in figure 6.12.

```
_ Tux.hs .
    createNeck = preservingMatrix$do
87
     transl 0 0.9 0.07
88
     createHead
89
     rotateY 90
90
     blackPenguin
^{91}
     sphere 0.8 0.45 0.5 0.45
^{92}
     transl 0 (-0.08) 0.35
93
     whitePenguin
^{94}
```

95	sphere 0.66 0.8 0.9 0.7
96	
97	<pre>createHead = preservingMatrix\$do</pre>
98	transl 0 0.3 0.07
99	createBeak
100	createEyes
101	rotateY 90
102	blackPenguin
103	sphere 1 0.42 0.5 0.42
104	
105	createBeak = do
106	preservingMatrix\$do
107	transl 0 (-0.205) 0.3
108	rotateX 10
109	beakColour
110	sphere 0.8 0.23 0.12 0.4
111	preservingMatrix\$do
112	beakColour
113	transl 0 (-0.23) 0.3
114	rotateX 10
115	sphere 0.66 0.21 0.17 0.38

Eyes

	Tux.hs
116	createEyes = preservingMatrix\$do
117	leftEye
118	leftIris
119	rightEye
120	rightIris
121	
122	leftEye = preservingMatrix\$do
123	transl 0.13 (-0.03) 0.38
124	rotateY 18
125	rotateZ 5
126	rotateX 5
127	whitePenguin
128	sphere 0.66 0.1 0.13 0.03
129	
130	rightEye = preservingMatrix\$do
131	transl (-0.13) (-0.03) 0.38
132	rotateY (-18)
133	rotateZ (-5)
134	rotateX 5
135	whitePenguin
136	sphere 0.66 0.1 0.13 0.03
137	
138	leftIris = preservingMatrix\$do
139	transl 0.12 (-0.045) 0.4

140	rotateY 18
141	rotateZ 5
142	rotateX 5
143	irisColour
144	sphere 0.66 0.055 0.07 0.03
145	
146	rightIris = preservingMatrix\$do
147	transl (-0.12) (-0.045) 0.4
148	rotateY (-18)
149	rotateZ (-5)
150	rotateX 5
151	irisColour
152	sphere 0.66 0.055 0.07 0.03

Legs

____ Tux.hs _ leftArm = preservingMatrix\$do 153rotateY 180 154transl (-0.56) 0.3 0 155rotateZ 45 156rotateX 90 157leftForeArm 158blackPenguin 159sphere 0.66 0.34 0.1 0.2 160 161 rightArm = preservingMatrix\$do 162transl (-0.56) 0.3 0 163 rotateZ 45 164rotateX(-90) 165rightForeArm 166blackPenguin 167sphere 0.66 0.34 0.1 0.2 168169 leftForeArm = preservingMatrix\$do 170transl (-0.23) 0 0 171rotateZ 20 172rotateX 90 173leftHand 174blackPenguin 175sphere 0.66 0.3 0.07 0.15 176177 rightForeArm = leftForeArm 178179leftHand = preservingMatrix\$do 180 transl (-0.24) 0 0 181 rotateZ 20 182rotateX 90 183 blackPenguin 184

```
sphere 0.5 0.12 0.05 0.12
185
186
    leftTigh = preservingMatrix$do
187
      rotateY 180
188
       transl (-0.28) (-0.8) 0
189
      rotateY 110
190
       leftHipBall
191
       leftCalf
192
193
       rotateY (-110)
194
       transl 0 (-0.1) 0
195
      beakColour
196
197
       sphere 0.5 0.07 0.3 0.07
198
    leftHipBall = preservingMatrix$do
199
      blackPenguin
200
       sphere 0.5 0.09 0.18 0.09
201
202
    rightTigh = preservingMatrix$do
203
       transl (-0.28) (-0.8) 0
204
      rotateY (-110)
205
      rightHipBall
206
      rightCalf
207
208
       transl 0 (-0.1) 0
209
       beakColour
210
       sphere 0.5 0.07 0.3 0.07
211
212
213
    rightHipBall = preservingMatrix$do
      blackPenguin
214
       sphere 0.5 0.09 0.18 0.09
215
216
    leftCalf = preservingMatrix$do
217
     transl 0 (-0.21) 0
218
     rotateY 90
219
     leftFoot
220
     beakColour
221
     sphere 0.5 0.06 0.18 0.06
222
223
    rightCalf = preservingMatrix$do
224
     transl 0 (-0.21) 0
225
     rightFoot
226
     beakColour
227
     sphere 0.5 0.06 0.18 0.06
228
```

Feet

 229
 foot = preservingMatrix\$do

 230
 scal 1.1 1.0 1.3

_ Tux.hs _

```
beakColour
231
232
       footBase
       toe1
233
       toe2
234
       toe3
^{235}
236
    footBase = preservingMatrix$do
237
       sphere 0.66 0.25 0.08 0.18
238
239
     toe1 = preservingMatrix$do
^{240}
       transl (-0.07) 0 0.1
241
       rotateY 30
242
^{243}
       scal 0.27 0.07 0.11
       createSphere 0.66
244
245
     toe2 = preservingMatrix$do
246
      transl (-0.07) 0 (-0.1)
247
      rotateY (-30)
^{248}
      sphere 0.66
                    0.27 0.07 0.11
249
250
    toe3 = preservingMatrix$do
251
       transl (-0.08) 0 0
252
       sphere 0.66 0.27 0.07 0.10
253
254
    leftFoot = preservingMatrix$do
255
       transl 0 (-0.09) 0
256
       rotateY (100)
257
       foot
258
259
    rightFoot = preservingMatrix$do
260
       transl 0 (-0.09) 0
261
       rotateY 180
262
263
       foot
```

The resulting image can be found in figure 6.13.

Tail

```
Tux.hs ______

264 createTail = preservingMatrix$ do

265 transl 0 (-0.4) (-0.5)

266 rotateX (-60)

267 transl 0 0.15 0

268 blackPenguin

269 sphere 0.5 0.2 0.3 0.1
```

The complete penguin

We can use all the parts to define Tux.

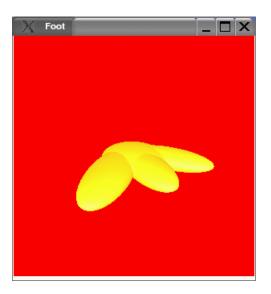


Figure 6.13: A penguin foot.

	Tux.hs
270	tux = preservingMatrix\$do
271	scale 0.35 0.35 (0.35::GLfloat)
272	rotateY (-180)
273	rotateZ (-180)
274	createTorso
275	createShoulders
276	createNeck
277	leftTigh
278	rightTigh
279	createTail

The resulting window can be found in figure 6.14.

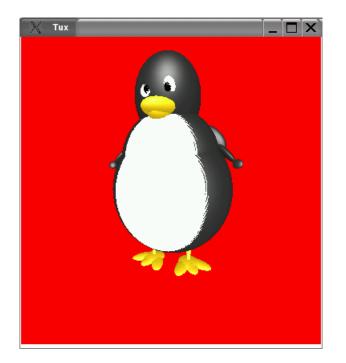


Figure 6.14: Tux the penguin..

Haskell Examples

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